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ALLOWABLE FLAW GEOMETRIES IN IRRADIATED Zr-2.5% Nb PRESSURE TUBES UNDER SLAR AND NORMAL OPERATING CYCLIC STRESSES

Report No. 87-292-K

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ABSTRACT

The assessment of blunt flaws in Zr-2.5% Nb pressure tubes involves the prevention of crack initiation from the flaw due to mechanical fatigue with subsequent crack propagation. Delayed hydride cracking may occur at even relatively shallow cracks in pressure tubes. In this report the allowable minimum root radii of postulated U-shaped flaws in the bulk of irradiated Zr-2.5% Nb pressure tubes are calculated such that crack initiation due to mechanical fatigue will not occur under SLAR (Spacer Location And Repositioning) and normal operating cyclic stresses.

The material fatigue life curve was estimated from data generated from experiments performed at AECL-CRNL on notched ring specimens taken from an irradiated Zr-2.5% Nb pressure tube. A safety factor of 20 on the number of cycles was placed on the "best fit" experimental curve, which resulted in a margin of 10 relative to the lower-bound experimental data. Flaws detected during inservice inspection of Pickering Unit 5 in 1987 are well within the allowable root radii guidelines given in this report.

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EXECUTIVE SUMMARY

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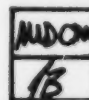
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The assessment of blunt flaws in Zr-2.5% Nb pressure tubes involves the prevention of crack initiation from the flaw due to mechanical fatigue with subsequent crack propagation. Delayed hydride cracking may occur at even relatively shallow cracks in pressure tubes. In this report the allowable minimum root radii of postulated U-shaped flaws in the bulk of irradiated Zr-2.5% Nb pressure tubes are calculated such that crack initiation due to mechanical fatigue will not occur under SLAR (Spacer Location And Repositioning) and normal operating cyclic stresses.

The fatigue analysis was based on a curve of the function (peak stress) x (alternating strain) versus cycles to crack initiation. The material fatigue life curve was estimated from data generated from experiments performed at AECL-CRNL on notched ring specimens taken from an irradiated Zr-2.5% Nb pressure tube. A safety factor of 20 on the number of cycles to crack initiation was placed on the "best fit" experimental curve in calculating the allowable root radii of postulated flaws in the pressure tube. This resulted in a safety margin of 10 relative to the lower-bound experimental data. Stresses due to the SLAR operation were obtained from AECL-Candu Operations. Flaws detected during inservice inspection of Pickering Unit 5 in 1987 are well within the allowable root radii guidelines given in this report.

It was recommended that fatigue tests conducted for the purpose of producing fatigue life curves for irradiated Zr-2.5% Nb pressure tube material be performed on smooth rather than notched specimens. It was also recommended that the calculation of cyclic strains in irradiated Zr-2.5% Nb under fully elastic-plastic conditions be further investigated.



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**ALLOWABLE FLAW GEOMETRIES IN IRRADIATED
Zr-2.5% Nb PRESSURE TUBES UNDER SLAR
AND NORMAL OPERATING CYCLIC STRESSES**

1.0 INTRODUCTION

The SLAR (Spacer Location And Repositioning) project involves repositioning the garter-spring spacers between the pressure tube and calandria tube to prevent contact between the two tubes. A schematic diagram of the SLAR tool is given in Figure 1-1. The SLAR operation involves subjecting the pressure tube to cyclic stresses much higher than normal operating stresses. See references /1-4/. The initiation of cracks at any surface flaw inside the pressure tube due to mechanical fatigue and subsequent propagation must be prevented. Elongated cracks of a depth of only about 0.25 mm will result during normal operation in a crack-tip stress intensity factor of 4.5 MPa $\sqrt{\text{m}}$, which is the lower-bound value at which crack growth due to delayed hydride cracking may occur/5/.

This report presents the minimum allowable root radii, as a function of flaw depth, of postulated U-shaped flaws in the bulk of the pressure tube such that crack initiation due to mechanical fatigue will not occur. Tables of allowable root radii are given for pressure tubes to be subjected to the SLAR operation plus normal operating cyclic stresses, and for pressure tubes to be subjected to normal operating cyclic stresses only.

Use is made of fatigue test results from AECL-CRNL, generated by testing specimens taken from an irradiated pressure tube taken from Pickering Unit 4 channel N16/6/. In this report low-cycle fatigue life curves are estimated from the test results. The curves are then applied to determine minimum allowable flaw root radii. The allowable flaw root radii incorporate a safety factor of 20 on the number of cycles to crack initiation.

Section 2 of this report contains fundamental concepts of low cycle fatigue. In Section 3 the low-cycle fatigue curves are estimated from the experimental data. In Section 4 the allowable flaw root radii are generated for various flaw depths. Conclusions and recommendations for further work on prevention of fatigue crack initiation in irradiated Zr-2.5% Nb pressure tubes are given in Section 5. A computer program was written to perform the analyses described in this report, and is available for future evaluations.

This analysis was funded by the Equipment and Reliability Department, Central Nuclear Services (Project Record Form 83-208).

2.0 BASIC THEORY OF LOW-CYCLE FATIGUE

The number of stress cycles predicted to occur during the SLAR operation and subsequent normal operating conditions is predicted to be 2000 cycles or less. This number of cycles falls into the category of low-cycle fatigue.

2.1 Fatigue Crack Initiation Curves

The familiar fatigue design curve of alternating stress versus cycles to crack initiation (or S-N curve) can be written in an equivalent form of alternating elastic strain versus cycles to crack initiation./7/

$$\bar{\epsilon}_{ae} = \frac{\sigma'_f}{E} (2N_1)^b \quad (2.1)$$

where $\bar{\epsilon}_{ae}$ = effective alternating elastic strain
 σ'_f = fatigue-strength coefficient, = true stress at fracture
 b = fatigue-strength exponent, usually $-0.12 \leq b \leq -0.05$
 E = Young's modulus
 N_1 = number of cycles to crack initiation.

The effective alternating strains are equal to one-half of the effective value of the range in strain components over a cycle. See equation (A-2) in Appendix A of this report. Equation (2.1) is plotted on log-log scales in Figure 2-1. A discussion on the applicability of using effective alternating strains as a crack initiation criterion is given in Appendix A.

The Coffin-Manson relation between the effective alternating plastic strain and cycles to crack initiation is:/24,25/

$$\bar{\epsilon}_{ap} = \epsilon'_f (2N_1)^c \quad (2.2)$$

where $\bar{\epsilon}_{ap}$ = effective alternating plastic strain,
 ϵ'_f = fatigue-ductility coefficient, = true strain at fracture,
 c = fatigue-ductility exponent, usually $-0.7 \leq c \leq -0.5$

Equation (2.2) is also plotted in Figure 2-1.

The effective total alternating strain can be written approximately as

$$\bar{\epsilon}_a = \bar{\epsilon}_{ae} + \bar{\epsilon}_{ap} \quad (2.3)$$

(total) (elastic) (plastic)

Upon substituting equations (2.1) and (2.2) into (2.3) one has the effective total strain versus cycles to crack initiation.

$$\bar{\epsilon}_a = \frac{\sigma'_f}{E} (2N_1)^b + \epsilon'_f (2N_1)^c \quad (2.4)$$

Equation (2.4) is plotted in Figure 2-1. From Figure 2-1, the elastic strains dominate at the high number of cycles, while the plastic strains dominate for the low number of cycles.

The two curves given by equations (2.1) and (2.2) are usually generated by performing experiments on smooth specimens with zero mean stress, and measuring sets of effective alternating elastic and plastic strains versus cycles to crack initiation. The data sets are plotted on the log-log scales shown in Figure 2-1. Straight lines are least-squares fitted to the elastic and plastic points to determine the material constants σ'_f , b and ϵ'_f , c . The fatigue experiments should be performed under strain controlled conditions. Strain controlled means that the magnitude of alternating strain during the experiment is held fixed.

2.2 Effect of Mean Stress

In application of the fatigue curves, the reduction in fatigue life from a tensile mean stress must be taken into account. The stress-strain function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ will be used for this purpose, where $\bar{\sigma}_{\max}$ is the peak effective tensile stress over the cycle, $\bar{\epsilon}_a$ is the effective alternating strain and E is Young's modulus. The uniaxial version of this function was proposed by Smith, Watson and Topper/26/, and was used to successfully reduce fatigue life data from many tests with different ratios of minimum stress/maximum stress down to a single curve. The materials tested were carbon steel and aluminum. This function was also used successfully in/12/ to correlate fatigue data results from tests performed on zirconium with a mean stress.

Equation (2.4) is modified to express $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ rather than $\bar{\epsilon}_a$ as a function of cycles to crack initiation/26/. Rewrite the elastic equation (2.1) in terms of alternating stress.

$$\bar{\sigma}_a = \sigma'_f (2N_i)^b \quad (2.5)$$

At a given life, the value of $\bar{\sigma}_a \bar{\epsilon}_a$ for a fully reversed test (zero mean stress) is assumed to be equal to $\bar{\sigma}_{\max} \bar{\epsilon}_a$ for a test with a non-zero mean stress. Upon substituting equation (2.5) into (2.4) the fatigue life equation which includes the effect of mean stress is written as:

$$(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2} = [(\sigma'_f)^2 (2N_i)^{2b} + \sigma'_f \epsilon'_f E (2N_i)^{b+c}]^{1/2} \quad (2.6)$$

The same material constants σ'_f , b , ϵ'_f and c described in Section 2.1 are still used to describe the fatigue life curve.

3.0 GENERATION OF FATIGUE LIFE CURVE FROM EXPERIMENTAL DATA

3.1 Experiments

The fatigue crack initiation experiments were performed at AECL-CRNL. Tests were performed on 10 mm wide ring specimens cut from an irradiated Zr-2.5% Nb pressure tube taken from Pickering Unit 4 channel N16. The pressure tube had

been operated for 82,370 full-power hours. Fast neutron irradiation was estimated at 6.77×10^{25} n/m²/6/. V-notches with various root radii (see Figure A-4 in Appendix A) were machined on the inside surface of the rings. The rings were then cycled in an air environment at room temperature. Two opposing point loads were applied through a zero to maximum to zero load cycle. Details are given in reference /6/. The experimental data obtained from AECL is given in Table 3-1.

3.2 Material Tensile Properties

The material's stress/strain properties are required in the calculations. Young's modulus was taken from reference /10/ to be 97×10^3 MPa, and Poisson's ratio was assumed to be 0.37. Static uniaxial stress/strain properties were taken from data measured in the circumferential direction from specimens cut from mid-way along a pressure tube removed from channel P12 of Bruce Unit 2/11/. The stress/strain curves were idealized in this report by using the Ramberg-Osgood equation. See Appendix A. A summary of all the stress/strain material properties used in this report are given in Table 3-2. The above-mentioned properties are the "20°C - Initial Static Value" properties.

Cyclic stress/strain curves for irradiated Zr-2.5% Nb pressure tube material are not available. The stress/strain curve for irradiated cold-worked Zr-2.5% Nb is expected to exhibit significant cyclic softening (lowering of the stress/strain curve) for the following reasons:

- a) Experiments performed on unirradiated cold-worked zirconium alloys have exhibited approximately 20% reduction in stress values after cyclic loading/12/.
- b) Radiation-induced dislocations are annihilated during plastic deformation at strain levels greater than about $1\frac{1}{2}$ /27,28/. The annihilation occurs in swaths or channels through the microstructure. The lower dislocation density in these channels allows plastic slip to occur at stress levels which are much lower than in the remaining material, and are close to stress levels associated with unirradiated material.

The stabilized cyclic version of the material's stress/strain curve was therefore taken from tensile data generated from unirradiated Zr-2.5% Nb pressure tube material/29/. This data is also given in Table 3-2, under the category "20°C - Stabilized". The static and stabilized cyclic material stress/strain curves are plotted in Figure 3-1. It was assumed that the material's cyclic stress/strain curve stabilized after 5 cycles.

3.3 Calculation of Stresses and Alternating Strains

The effective alternating strains are equal to one-half of the effective value of the range in strain components over a cycle. See equation (A-2) in Appendix A of this report. The experimental data was converted to sets of notch-tip effective alternating elastic strains, notch-tip effective alternating plastic strains, and notch-tip peak stress, versus cycles to crack

initiation. This involved modelling the cyclic history of the tri-axial stress/strain components at the notch root, and then converting the stress and strain components to effective stress and strain. Details of the calculations are given in Appendix A. The calculation of nominal stresses in the experiments is outlined in Appendix B of this report. It is assumed that an analysis based on isotropic material behaviour is adequate. It is also assumed that the conditions at the notch root are plane strain.

The notch-tip elastic tensile stress concentration factor is based on the nominal inside wall stress:

$$k_t = \frac{\text{notch-tip vertical stress}}{\text{nominal inside-wall stress}} \quad (3.1)$$

Values are given in Table 3-3. Details of the calculation of the stress concentration factors are given in Appendix A. Also given in Table 3-3 are the stabilized effective alternating elastic and plastic strain, the effective peak total strain for the cycle, and the effective mean and peak stress. In all but specimens 9 and 10 the effective alternating plastic strains are of the same order of magnitude or greater than the effective alternating elastic strains. Although non-zero mean stress values were calculated, it was conservatively assumed that the mean stresses actually relaxed to zero (see reference /12/) due to plastic slip in the presence of cyclic plastic straining, which is not accounted for in the constitutive model. The result is lower values of peak stress, $\bar{\sigma}_{\max}$, and an overall lower fatigue life curve as expressed in terms of the function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ versus cycles to crack initiation.

Values of the elastic function $(\bar{\sigma}_{\max} \bar{\epsilon}_{ae} E)^{1/2}$ are plotted against cycles to crack initiation in Figure 3-3, values of the plastic function $(\bar{\sigma}_{\max} \bar{\epsilon}_{ap} E)^{1/2}$ are plotted against cycles to crack initiation in Figure 3-4, and values of the elastic-plastic function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ are plotted in Figure 3-5.

3.4 Material's Fatigue Life Curve

From Table 3-3, the calculated peak effective total strains for the first four specimens are in the range of 11 to 14%. It is proposed that, due to the very high strains, some small cracks had initiated at the notch root prior to the initiation time detected by the instrumentation in the experiment. For this reason the first four specimens were excluded from the data set used to correlate the fatigue life curves. Specimens 9 and 10 did not initiate cracks after 10^6 cycles, at which point the tests were stopped. It will be conservatively assumed that crack initiation did occur in these two specimens at 10^6 cycles, but specimens 9 and 10 will be used only in determining the elastic parameters σ'_f and b .

Straight lines were fit by using least-squares to the log-log plots of the elastic fatigue data in Figure 3-3, and to the plastic fatigue data in Figure 3-4. The resultant material fatigue properties are:

fatigue strength coefficient σ'_f = 749 MPa

fatigue strength exponent b = -0.0885

fatigue ductility coefficient ϵ'_f = 0.642

fatigue ductility exponent c = -0.538

All of the fatigue properties fall in the range of generally accepted values. The σ'_f should be an approximation of the material's true fracture stress. The properties ϵ'_f and c agree well with representative values determined experimentally for unirradiated Zircaloy-2 and Zr-2.5% Nb by Hosbons (AECL-CRNL)/30,31/ of ϵ'_f = 0.80 and c = -0.48. The lower ductility coefficient for the irradiated material at 20°C compared with the unirradiated material at 300°C is a reasonable trend.

4.0 ALLOWABLE FLAW FOOT RADII FOR SLAR AND NORMAL OPERATING CYCLIC STRESSES

Allowable minimum flaw root radii were determined for postulated U-shaped flaws of various depths in the bulk of the pressure tube for SLAR plus normal operating cyclic stresses, and for normal operating cyclic stresses only.

4.1 Pressure Tube Dimensions

The dimensions are for a crept pressure tube in Pickering 'B' after 15 years of service/14/.

inner radius = 52.2 mm

wall thickness = 3.91 mm

4.2 Pressure Tube Nominal Stresses

4.2.1 Stresses Due to the SLAR Operation

Details of the stresses induced by the SLAR tool are given in references /1-4/. Stresses are induced by the contact of the three saddles with the top of the tube, and by contact of the bearing with the bottom of the tube. The separation distance of about 300 mm between the saddles and each bearing is sufficient to allow the axial profiles of stress due to the saddles and the bearing to be treated separately.

The maximum hoop stress on the inside wall occurs at the 3 o'clock and 9 o'clock positions on the tube. Stresses at this angular location were used to generate the allowable flaw sizes. Values of cyclic nominal stress and number of cycles are given in Table 4-1. The nominal stresses follow a zero to maximum to zero cycle. Details of the idealization of the nominal stresses and number of cycles are given in Appendix B of this report. For reasons given in Appendix B, stresses calculated by AECL/1,2,4/ for a 30" bearing crown radius were used, even though the bearing crown radius was increased to 80" to reduce peak stresses. The stresses also incorporate a factor of 1.3

times the calculated finite element results, as recommended in reference /2/.

4.2.2 Stresses Due to Operating Pressure

The cyclic nominal stress and number of cycles are given in Table 4-1. Details of the calculations are given in Appendix B. The stresses are based on an operating pressure of 10.0 MPa. The number of cycles is 550 startup/shutdown cycles.

4.3 Material Properties Used in Allowable Root Radii Calculations

4.3.1 Tensile Properties

The SLAR operation is performed at room temperature. The material's static and stabilized stress/strain properties used in the SLAR portion of the fatigue modelling are the room temperature properties given in Table 3-2 and Figure 3-1. The temperature range during normal operation is bounded by 20°C and 270°C. The properties at 270°C will be used to predict the normal operating portion of the cyclic response history. The 270°C properties will result in higher predicted strains, due to a lower stress/strain curve, and hence result in an added conservatism.

Young's modulus at 270°C was taken from /10/ to be 80×10^3 MPa, Poisson's ratio was taken to be -0.40 , and the static elastic-plastic stress/strain properties were taken from /11/. Following the same reasoning as given in Section 3.2, the stabilized cyclic elastic-plastic stress/strain properties were taken from results given for unirradiated material at 270°C in /29/. The tensile properties at 270°C are summarized in Table 3-2. The static and stabilized cyclic stress/strain curves are given in Figure 3-2.

4.3.2 Fatigue Properties

A safety factor of 20 on the number of cycles was imposed on the "best fit" fatigue life curves for the purposes of calculating allowable flaw root radii. The resulting elastic, plastic and elastic-plastic fatigue life curves incorporating this safety factor are plotted in Figures 3-3 to 3-5 respectively. The material's fatigue properties adjusted for a safety factor of 20 on the number of cycles are:

fatigue strength coefficient $\sigma'_f = 574$ MPa

fatigue strength exponent $b = -0.0885$

fatigue ductility coefficient $\epsilon'_f = 0.128$

fatigue ductility exponent $c = -0.538$

It will be shown in Section 4.4 that the use of these fatigue properties will result in a safety margin of about 10 on the lower-bound experimental data for all the allowable flaw sizes calculated in this report. One purpose of the imposed safety factor is to attempt to account for surface irregularities in a notch.

4.4 Allowable Flaw Root Radii

It is assumed that the flaw is in the shape of a U-notch, illustrated in Figure 4-1. The details of the calculation of the elastic notch-tip stress concentration factor are given in Appendix A. The values of effective stresses and effective alternating elastic and plastic strains were calculated as described in Section 3.3 and Appendix A. The calculation of the allowable root radius for a given flaw depth followed an iterative procedure:

- i) For a given estimate of the allowable root radius, the effective stresses and effective alternating strains are calculated for each cyclic load case.
- ii) The number of cycles to crack initiation is determined for each value of the function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ from the material's fatigue life curve.
- iii) The fatigue usage factor is calculated, including the safety factor of 20 on cycles to crack initiation.
- iv) The allowable root radius is adjusted to result in a usage factor of 1.0.

4.4.1 Allowable Flaw Root Radii for SLAR Plus Normal Operating Cyclic Stresses

The minimum allowable root radii for SLAR plus normal operating cyclic stresses for flaw depths up to 1.0 mm are given in Table 4-2. A root radius of 0.05 mm or less should be treated as a crack rather than a smooth notch, and should be evaluated by using fracture mechanics. The root radii below this threshold are marked by asterisks in Table 4-2.

The stress across the remaining ligament increases as the flaw depth increases. To compensate for this the deeper flaws are blunter than the shallow flaws. The net result is that the elastic notch-tip stress concentration factor for SLAR stresses remains essentially constant for all flaw depths. This is seen in Tables 4-3 and 4-4. These tables contain, for each of the five load cases, values of elastic notch-tip stress concentration factor, and stabilized values of effective alternating elastic and plastic strains, effective mean stress, effective peak stress and the fatigue usage factor. Table 4-3 contains data for the postulated 0.10 mm deep flaw, and Table 4-4 contains data for the postulated 1.0 mm deep flaw. The quantities vary smoothly for intermediate flaw depths.

In calculating the data points from the experiments, it was assumed that the mean stresses relaxed to zero. Conversely, it is conservatively assumed that the mean stresses calculated for the postulated flaws in the pressure tube do not relax. This is due to the lower number of stress cycles in the pressure tube. The result is higher values of the function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ compared with a zero mean stress being used. Slightly larger root radii for a given flaw depth are therefore calculated with a non-zero mean stress.

The usage factors for the load cases "Bearing-High Stress", "Saddles" and "Operating Pressure" sum to essentially 1.0, and therefore govern in

determining the allowable flaw root radii. The values of the stress function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ for these three load cases are plotted on the fatigue life curve (which includes a safety factor of 20 on the number of cycles) in Figure 3-5. A safety margin of about 10 on number of cycles exists for these points over the lower-bound experimental data.

In Figure 4-2 is plotted the variation of the minimum allowable flaw root radius with flaw depth. The figure is divided into three regions. The region to the left of the vertical line at a root radius of 0.05 mm is for flaws which should be treated as cracks and evaluated by using fracture mechanics. The regions to the right of the vertical line are for flaws which are acceptable or unacceptable following the criteria in this report. Also plotted are points corresponding to flaws found during in-service inspection of Pickering Unit 5/16/. The root radii of these flaws was assumed to be one-half the flaw width. A large margin exists on the width of these flaws.

4.4.2 Allowable Flaw Root Radii for Normal Operating Cyclic Stresses Only

The minimum allowable root radii for normal operating cyclic stresses only are also given in Table 4-2, with values below 0.05 mm marked by asterisks. The notch-tip parameters are now the same for all flaws and are given in Table 4-5. Due to the lower stress from normal operating pressure compared with SLAR induced stresses, the allowable root radii are much sharper. The value of the stress function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)^{1/2}$ is plotted on Figure 3-5.

In Figure 4-3 the variation of the allowable minimum flaw root radius with flaw depth is plotted. As in Figure 4-2, the figure is divided into three regions; for flaws which are acceptable, unacceptable, or should be evaluated as cracks by using fracture mechanics.

4.5 Quality Assurance of Allowable Flaw Root Radii

The objective of this report is to produce conservative values of minimum allowable flaw root radii to prevent crack initiation due to mechanical fatigue. The following steps were taken to ensure the reliability of the values given in Table 4-2, and Figures 4-2 and 4-3.

- a) The elastic notch-tip stress concentration factors for the V-notch (experiment) and the U-notch (application to the pressure tube) were found to produce comparable values. These results were also found to be in reasonable agreement with elastic notch-tip stress concentration factors based on the crack-tip stress intensity factor. See reference /17/.
- b) The testing of specimens 9 and 10 was stopped prior to crack initiation, at 10^6 cycles. It was conservatively assumed that crack initiation did occur in these two specimens at 10^6 cycles.
- c) The cyclic stress/strain calculations for the notched experimental specimens yielded material fatigue properties which were reasonable in magnitude relative to fatigue properties generated previously for

unirradiated zirconium alloys. This indicates that the calculated cyclic stress/strain values are a reasonable approximation of the actual stress/strain values.

- d) In calculating the material's fatigue life curve it was assumed that the mean stress at the notch root in all but specimens 9 and 10 in the experiments had relaxed to zero. The result was lower estimates of peak stress and a lower material fatigue life curve. In calculating the allowable flaw root radii in the pressure tubes it was assumed that no mean stress relaxation would occur, due to the lower number of cycles. The result was higher estimates of the applied peak stress at the notch root in the pressure tubes, compared with peak stresses which would have been obtained if the assumption of complete mean stress relaxation had been employed. The combined effect of assuming complete mean stress relaxation in the experiment and no mean stress relaxation for postulated flaws in the pressure tube resulted in an added conservatism in the calculations.
- e) The values of cycles to crack initiation taken from the fatigue life curve (which included a safety factor on number of cycles) used in determining allowable root radii were about a factor of 10 less than the lower-bound experimental data points.
- f) The fatigue usage factor calculations in the computer program were confirmed by hand calculation.

5.0 CONCLUSIONS AND RECOMMENDATIONS

- 1) If the root radius of a flaw in the bulk of an irradiated Zr-2.5% Nb pressure tube is equal to or greater than the corresponding value given in Table 4-2, and Figures 4-2 or 4-3, a crack will not initiate by mechanical fatigue due to SLAR and normal operating cyclic stresses.
- 2) Flaws detected during inservice inspection of Pickering Unit 5 in 1987 are well within the allowable root radii guidelines given in this report.
- 3) Fatigue tests conducted for the purpose of producing fatigue life curves for irradiated Zr-2.5% Nb pressure tube material should be performed on smooth specimens under strain control rather than notched specimens.
- 4) Fatigue life data obtained from smooth-specimen tests should be compared with data generated from notched-specimen tests to check the accuracy of the notch-root cyclic stress/strain calculations.
- 5) Cyclic material stress/strain curves should be generated from irradiated Zr-2.5% Nb pressure tube material for use in future fatigue evaluations.
- 6) The calculation of cyclic strains in irradiated Zr-2.5% Nb under fully elastic-plastic conditions should be further investigated. This includes an assessment of the effect of material anisotropy.

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REFERENCES

1. Mueller-Schabkar, A. and Villamagna, A., "SLAR - Interim Summary of Pressure Tube Stresses", AECL-Candu Operations, Report No. 40-31100-226-001, File No. 40-31100-226-001, June 18, 1985.
2. Wong, H., "Effect of SLAR Induced Stresses on Pressure Tubes", AECL-Candu Operations, Report No. 40-31100-200-024, December 5, 1986.
3. Memo from A. Schabkar to H. Wong, "SLAR Pressure Tube Hoop Stresses", AECL CANDU Operations File No. 40-31100-200-00, April 1, 1985.
4. Memo from H. Wong to R. Lior, "Maximum P/T Stresses on the Inside Surface Used in Crack Initiation Tests", AECL CANDU Operations File No. 40-31100-200-00, March 3, 1987.
5. Simpson, L.A. and Puls, M.P., Metallurgical Transactions, 10A, 1979, pp. 1093.
6. Wong, H., Draft report on assessment of irradiated fatigue test data on simulated SLAR stress cycles, AECL-Candu Operations, File No. 40-31100-200-034, 1987.
7. Mitchell, M.R., "Fundamentals of Modern Fatigue Analysis for Design", in "Fatigue and Microstructure", 1978 ASM Materials Science Seminar, St. Louis, Missouri, October 14-15, 1978, pp 385-437.
8. "Criteria of the ASME Boiler and Pressure Vessel Code for Design by Analysis in Section III", prepared by the ASME Section III Working Group on Plant Design, 1985.
9. Leis, B.N. and Topper, T.H., "Cyclic Deformation and Fatigue Analysis for Notched Components", Nuclear Engineering and Design, Vol. 29, 1974, pp. 370-383.
10. Memo from C.K. Chow to L.A. Simpson, "Young's Modulus of Zr-2.5% Nb Pressure Tube Material", AECL-WNRE, March 10, 1987.
11. Causey, A.R., Chow, C.K., Coleman, C.E., Ells, C.E. Fidleris, V., Ibrahim, E.F., Rodgers, D.K. and Urbanic, V.F., "The Examination of the Pressure Tube Removed from Channel P12 of Bruce Unit 2", AECL-CRNL, Report No. CRNL-4015, February, 1987.
12. Handfield, L., and Dickson, J.I., "The Low-Cycle Fatigue and Cyclic Behaviour of Zirconium", in "Advances in Fracture Research", Proceedings of the 5th International Conference on Fracture (ICF5), Cannes, France, March 29 - April 3, 1981, pp. 1411-1418.

13. Private communication with R.R. Hosbons, AECL-CRNL, July, 1987.
14. Memo from M. Natesan to D.D. Coleman, "Pressure Tube Data for Different Hydro Reactors", Ontario Hydro, September 9, 1986.
15. Dieter, G.E., "Mechanical Metallurgy", Second Edition, McGraw-Hill, 1976.
16. Pickering Unit 5 pressure tube inspection - preliminary results, June 1987.
17. Tada, H., Paris, P.C. and Irwin, G.R., "The Stress Analysis of Cracks Handbook", Del Research Corporation, St. Louis, Missouri.
18. Mendelson, A., "Plasticity: Theory and Application", The MacMillan Co., 1968.
19. Ohno, N. and Satra, M., "Detailed and Simplified Elastoplastic Analyses of a Cyclically Loaded Notched Bar", ASME Journal of Engineering Materials and Technology, Vol. 109, July, 1987, pp. 194-202.
20. Roark, R.J. and Young, W.C., "Formulas for Stress and Strain", Fifth Edition, McGraw-Hill, 1975.
21. Peterson, R.E., "Stress Concentration Factors", John Wiley and Sons, 1974.
22. Neuber, H., "Theory of Stress Concentration for Shear-Strained Prismatic Bodies with Arbitrary Nonlinear Stress-Strain Law", Transactions of the ASME, December, 1961, pp. 544-550.
23. Timoshenko, S.P. and Goodier, J.N., "Theory of Elasticity", Third Edition, McGraw-Hill, 1970.
24. Coffin, L.F., Jr., and Tavernelli, J.F., "The Cyclic Straining and Fatigue of Metals", Transactions of the Metallurgical Society, AIME, Vol. 215, Oct. 1959, pp. 794-806.
25. Manson, S.S., "Interfaces Between Fatigue, Creep and Fracture", International Journal of Fracture Mechanics, Vol. 2, 1966, pp. 327-363.
26. Smith, K.N., Watson, P. and Topper, T.H., "A Stress/Strain Function for the Fatigue of Metals", Journal of Materials, JMLSA, Vol. 5, No. 4, Dec. 1970, pp. 767-778.
27. Northwood, D.O., Gilbert, R.W. and Coleman, C.E., "Effect of Stress on Radiation Damage in Neutron Irradiated Zirconium Alloys", in "Zirconium in the Nuclear Industry", ASTM STP 633, 1977, pp. 418-436.

28. Hosbons, R.R., Coleman, C.E. and Holt, R.A., "Numerical Simulation of Tensile Behaviour of Nuclear Fuel Cladding Materials", presented at the 3rd International Conference on Structural Mechanics in Reactor Technology, London, England, Sept. 1-5, 1975, also AECL-CRNL Report No. 5245.
29. Moan, G.D., "A Survey of the Effect of Irradiation on the Tensile Properties of Zr-2.5% Nb Pressure Tubes", AECL-Candu Operations, Report No. TDVI 384, Dec., 1983.
30. Hosbons, R.R., "Low-Cycle Fatigue Behaviour of Zircaloy at 573 K", ASTM STP 520, 1973, pp. 482-489.
31. Hosbons, R.R., "Low-Cycle Fatigue Behaviour of Zirconium Alloys at 300°C", International Conference on Creep and Fatigue in Elevated Temperature Applications", Philadelphia, Sept., 1973, and Sheffield, U.K., April, 1974, pp. 176.1-176.6.
32. Miner, M.A., "Cumulative Damage in Fatigue", Journal of Applied Mechanics, Vol. 12, 1945, pp. A159-A164.

TABLE 3-1

Summary of AECL Fatigue Experimental Data

AECL Test Group	AECL Specimen Number	Notch Depth (mm)	Notch Root Radius (mm)	Wall Thickness (mm)	Nominal Inside Wall Stress (MPa)	Number of Cycles to Crack Initiation
I	1	1.15	0.12	3.78	310	835
I	2	1.38	0.12	4.29	310	475
I	3	1.28	0.12	4.29	310	602
I	4	1.24	0.12	4.51	310	541
II	5	1.15	0.50	4.14	310	2785
II	6**	1.18	0.50	4.42	310	Test Void
II	7*	1.13	0.50	4.19	310	1517
II	8*	0.92	0.50	4.19	310	2054
III	9**	1.07	0.50	4.09	100	>1000000
III	10**	1.09	0.50	4.27	100	>1000000
II	11	1.16	0.50	4.17	310	1451
II	12	1.12	0.50	4.34	310	1540
IV	13	1.24	0.12	4.15	149	1305
IV	14	1.23	0.12	4.30	149	1113
IV	15	1.00	0.12	4.30	149	1904
IV	16	1.11	0.12	4.11	149	1502
V	17	0.92	0.12	4.27	120	15226
V	18	0.82	0.12	4.17	120	45879
V	19*	0.84	0.12	4.34	120	7038
V	20*	1.10	0.12	4.34	120	4217
VI	21	0.82	0.12	4.11	310	850
VI	22	0.60	0.12	4.11	310	990
VI	23*	0.60	0.12	4.29	310	990
VI	24*	0.55	0.12	4.34	310	425

*Specimen thermally cycled before fatigue testing.

**Test data not complete.

TABLE 3-2

Tensile Properties of Zr-2.5% Nb Pressure Tube
Material Used in the Cyclic Stress/Strain Analysis

Quantity	20°C		270°C	
	Initial Static Value	Stabilized Value After 5 Cycles	Initial Static Value	Stabilized Value After 5 Cycles
Young's Modulus, E (MPa)	97×10^3	97×10^3	80×10^3	80×10^3
Poisson's Ratio, ν	0.37	0.37	0.40	0.40
Yield Stress (MPa)	1000	500	1000	450
True Stress at Fracture	1050	550	1050	500
True Strain at Fracture	0.13	0.13	0.13	0.13
Ramberg-Osgood ⁺ Constants:				
α	5.561	5.774	5.561	4.530
n	15.00	15.00	15.00	15.00

⁺See Appendix A of this report.

TABLE 3-3

Stabilized Cyclic Notch-Tip Conditions in Fatigue Experiments

AECL Specimen Number	Elastic Stress Concentration Factor k_t	Effective Alternating Elastic Strain $\bar{\epsilon}_{ae}$	Effective Alternating Plastic Strain $\bar{\epsilon}_{ap}$	Effective Peak Total Strain	Effective Mean Stress (MPa)	Effective ⁺ Peak Stress $\bar{\sigma}_{max}$ (MPa)
1	9.35	0.00446	0.0400	0.1094	*57.7	473
2	10.61	0.00460	0.0511	0.1376	*47.1	489
3	9.74	0.00450	0.0433	0.1178	*54.1	478
4	9.19	0.00444	0.0387	0.1061	*59.3	471
5	4.67	0.00370	0.00903	0.0324	* 127	393
7	4.58	0.00368	0.00864	0.0315	* 128	391
8	3.99	0.00350	0.00616	0.0255	* 129	372
9	4.32	0.00188	0.000087	0.0122	184	271
10	4.31	0.00188	0.000087	0.0122	183	271
11	4.69	0.00371	0.00911	0.0326	* 127	394
12	4.50	0.00366	0.00825	0.0306	* 128	389
13	9.45	0.00367	0.00845	0.0310	* 128	390
14	9.20	0.00364	0.00791	0.0297	* 129	386
15	7.66	0.00339	0.00503	0.0228	* 128	360
16	8.50	0.00354	0.00654	0.0264	* 130	376
17	7.17	0.00295	0.00238	0.0158	* 110	314
18	6.67	0.00283	0.00194	0.0145	* 104	301
19	6.72	0.00285	0.00198	0.0146	* 105	302
20	8.21	0.00317	0.00343	0.0187	* 120	337
21	6.79	0.00411	0.0209	0.0615	*91.9	436
22	5.67	0.00392	0.0141	0.0448	* 112	416
23	5.65	0.00392	0.0140	0.0446	* 112	416
24	5.42	0.00387	0.0128	0.0415	* 116	411

* Assume mean stress has relaxed to zero in determining fatigue life curves.

+ Based on relaxed mean stresses.

TABLE 4-1

Nominal Cyclic Stresses in Pressure Tube

Load Case	Nominal Peak ⁺ Inside Wall Hoop Stress (MPa)	Normalized ⁺⁺ Membrane Component of Hoop Stress	Normalized ⁺⁺ Bending Component of Hoop Stress	Nominal Peak ⁺ Longitudinal Membrane Stress (MPa)	Number of Cycles
Bearing-High Stress	305	0.500	0.500	90	32
Saddles	211	0.500	0.500	90	224
Bearing-Medium Stress	152	0.500	0.500	90	48
Bearing-Low Stress	90	0.500	0.500	45	112
Operating Pressure	139	0.964	0.036	67	550

+ Stress cycle is from zero to maximum value to zero.

++ Normalization is performed by dividing by the nominal inside wall stress.

TABLE 4-2

Allowable Root Radii of Postulated U-Notches in Pressure Tubes

Flaw Depth (mm)	SLAR Plus Normal Operating Stresses		Normal Operating Stresses Only	
	Allowable Root Radius (mm)	Radius/Depth	Allowable Root Radius (mm)	Radius/Depth
0.05	0.021*	0.413	0.0084*	0.168
0.10	0.042*	0.422	0.018*	0.177
0.15	0.065	0.431	0.028*	0.187
0.20	0.088	0.441	0.039*	0.196
0.25	0.113	0.452	0.052	0.206
0.30	0.139	0.463	0.065	0.216
0.35	0.166	0.474	0.079	0.226
0.40	0.195	0.487	0.095	0.237
0.45	0.226	0.501	0.111	0.247
0.50	0.258	0.516	0.129	0.259
0.55	0.293	0.533	0.149	0.271
0.60	0.330	0.550	0.170	0.283
0.65	0.371	0.570	0.193	0.296
0.70	0.414	0.592	0.217	0.310
0.75	0.462	0.616	0.244	0.326
0.80	0.515	0.644	0.274	0.342
0.85	0.573	0.675	0.306	0.360
0.90	0.639	0.710	0.341	0.379
0.95	0.713	0.751	0.381	0.401
1.00	0.798	0.798	0.425	0.425

* Below lower-bound threshold (≈ 0.05 mm) for characterizing the flaw as smooth, as opposed to a crack.

TABLE 4-3

Stabilized Cyclic Notch-Tip Conditions Due to
SLAR Plus Normal Operating Stresses for a
Postulated 0.10 mm Deep U-Notch With a Root Radius of 0.04 mm

Load Case	Elastic Stress Conc. Factor	Effective Alternating Elastic Strain, $\bar{\epsilon}_{ae}$	Effective Alternating Plastic Strain, $\bar{\epsilon}_{ap}$	Effective Mean Stress (MPa)	Effective Peak Stress, $\bar{\sigma}_{max}$ (MPa)	Fatigue Usage Factor
Bearing- High Stress	4.16	0.00365	0.00664	163	421	0.3920
Saddles	4.16	0.00308	0.00251	142	357	0.4607
Bearing- Medium Stress	4.16	0.00253	0.000894	124	296	0.0144
Bearing- Low Stress	4.16	0.00162	0.000157	78.5	189	0.0003
Operating Pressure	4.25	0.00293	0.00106	109	273	0.1326

TABLE 4-4

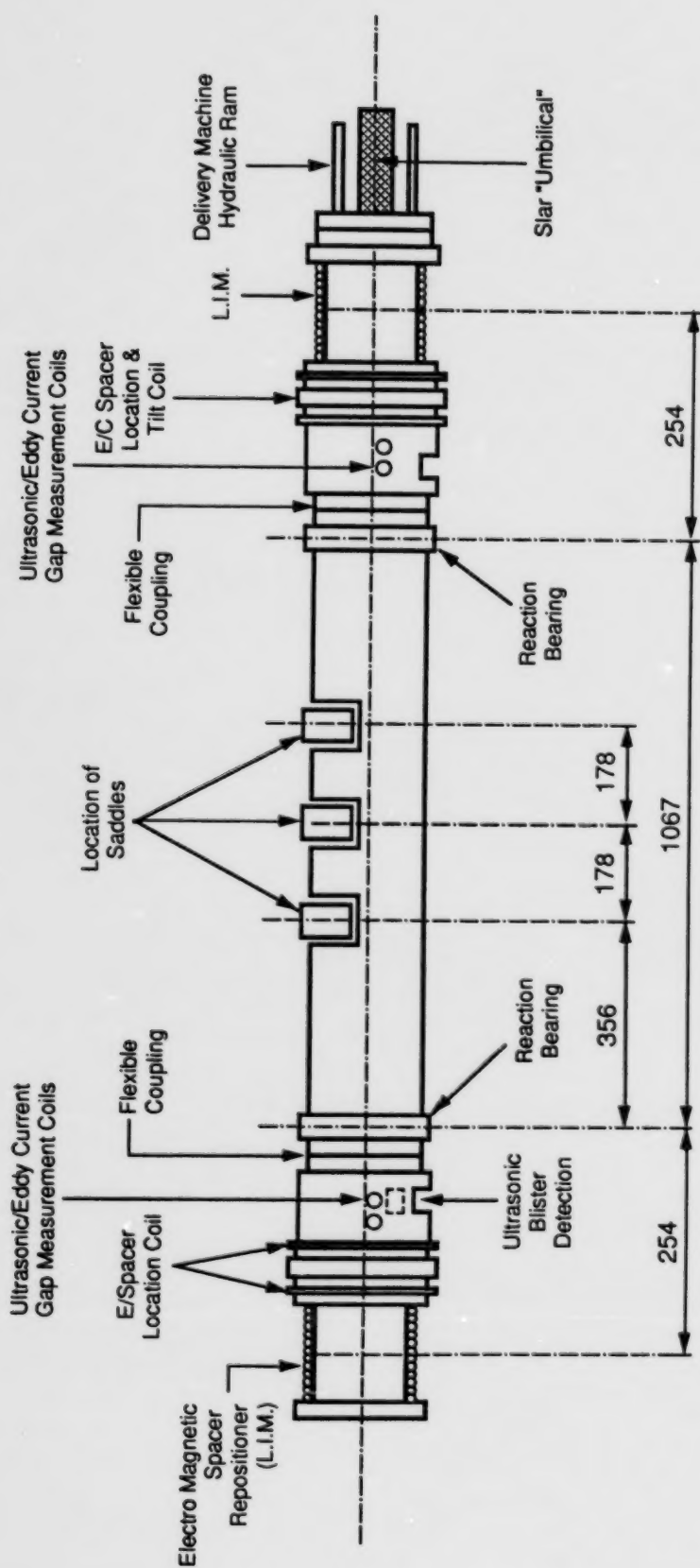
Stabilized Cyclic Notch-Tip Conditions Due to
 SLAR Plus Normal Operating Stresses for a
Postulated 1.0 mm Deep U-Notch With a Root Radius of 0.80 mm

Load Case	Elastic Stress Conc. Factor	Effective Alternating Elastic Strain, $\bar{\epsilon}_{ae}$	Effective Alternating Plastic Strain, $\bar{\epsilon}_{ap}$	Effective Mean Stress (MPa)	Effective Peak Stress, $\bar{\sigma}_{max}$ (MPa)	Fatigue Usage Factor
Bearing- High Stress	3.99	0.00359	0.00601	161	414	0.3304
Saddles	3.99	0.00302	0.00223	140	350	0.3721
Bearing- Medium Stress	3.99	0.00246	0.000778	122	289	0.0110
Bearing- Low Stress	3.99	0.00156	0.000137	78.5	184	0.0002
Operating Pressure	4.77	0.00314	0.00157	111	292	0.2863

TABLE 4-5

Stabilized Cyclic Notch-Tip Conditions
Due to Normal Operating Loads Only for
Postulated U-Notches of Depths Between 0.05 and 1.00 mm

Quantity	Value
Elastic Stress Conc. Factor	5.94
Effective Alternating Elastic Strain, $\bar{\epsilon}_{ae}$	0.00347
Effective Alternating Plastic Strain, $\bar{\epsilon}_{ap}$	0.00330
Effective Mean Stress	86.7 MPa
Effective Peak Stress, $\bar{\sigma}_{max}$	310 MPa
Fatigue Usage Factor	1.0000



all dimensions in mm

FIGURE 1-1
SCHEMATIC OF THE SLAR BENDING TOOL
(from reference 2)

KNARF DISK

175959-RD

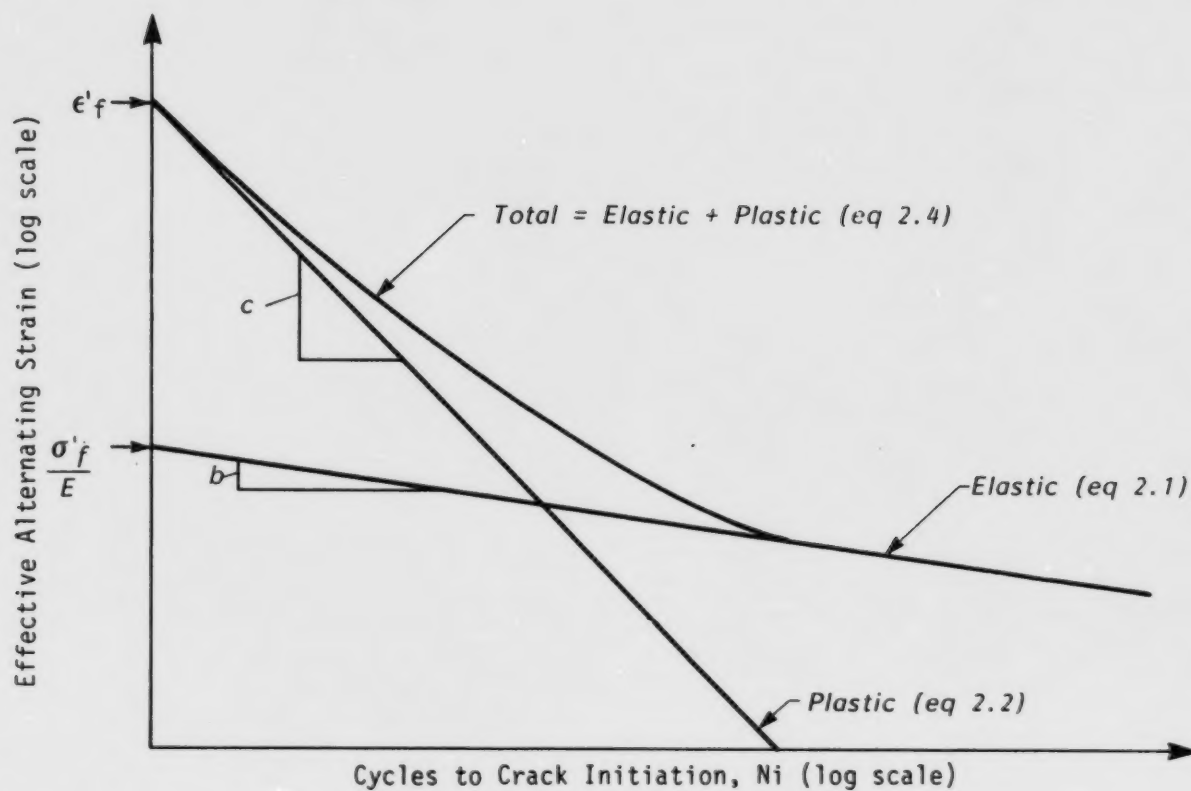


FIGURE 2.1
ILLUSTRATION OF FATIGUE DESIGN CURVES
(FROM REFERENCE/7/)

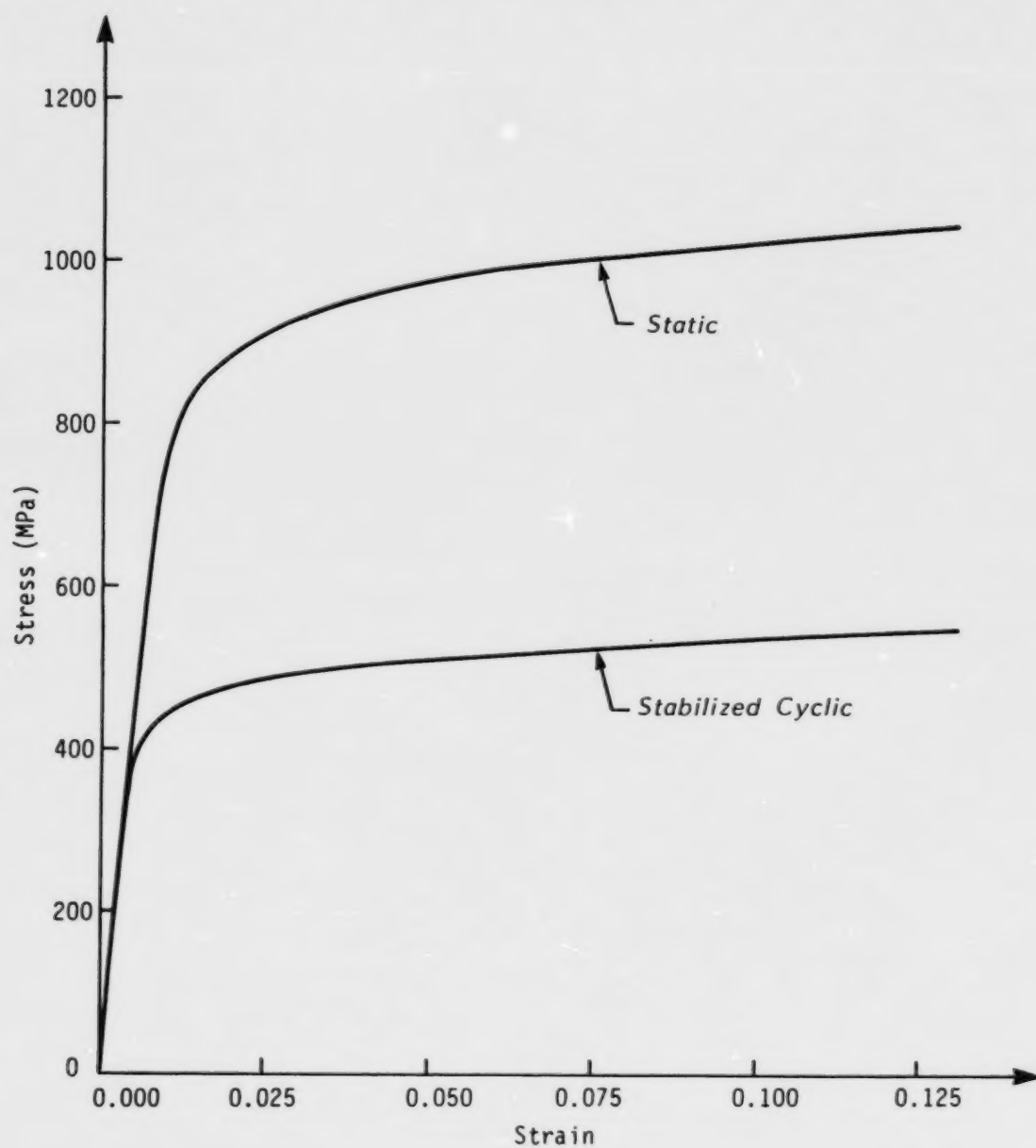


FIGURE 3-1
MATERIAL'S IDEALIZED UNIAXIAL STRESS/STRAIN
CURVES AT 20°C

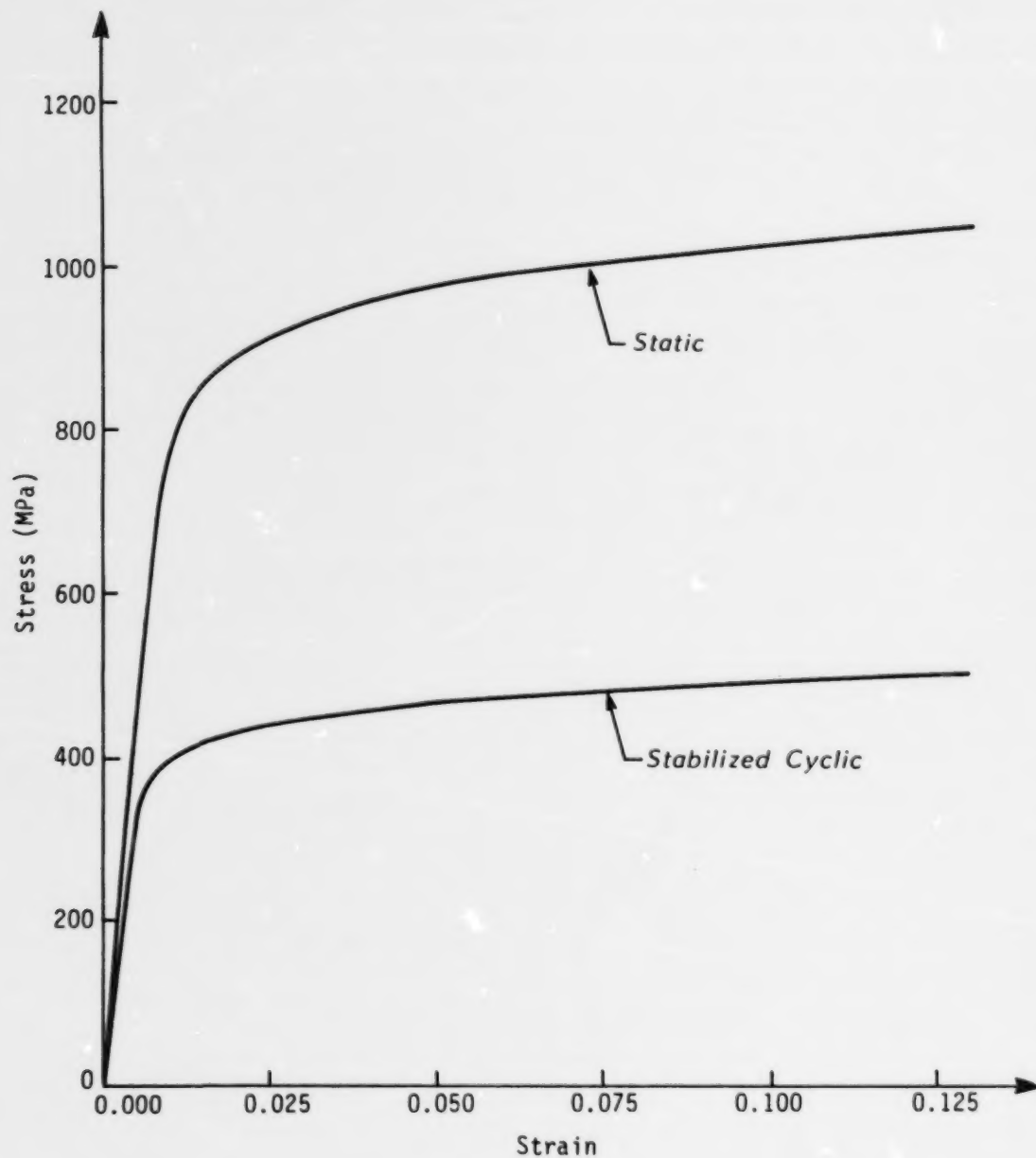


FIGURE 3-2
MATERIAL'S IDEALIZED UNIAXIAL STRESS/STRAIN
CURVES AT 270°C

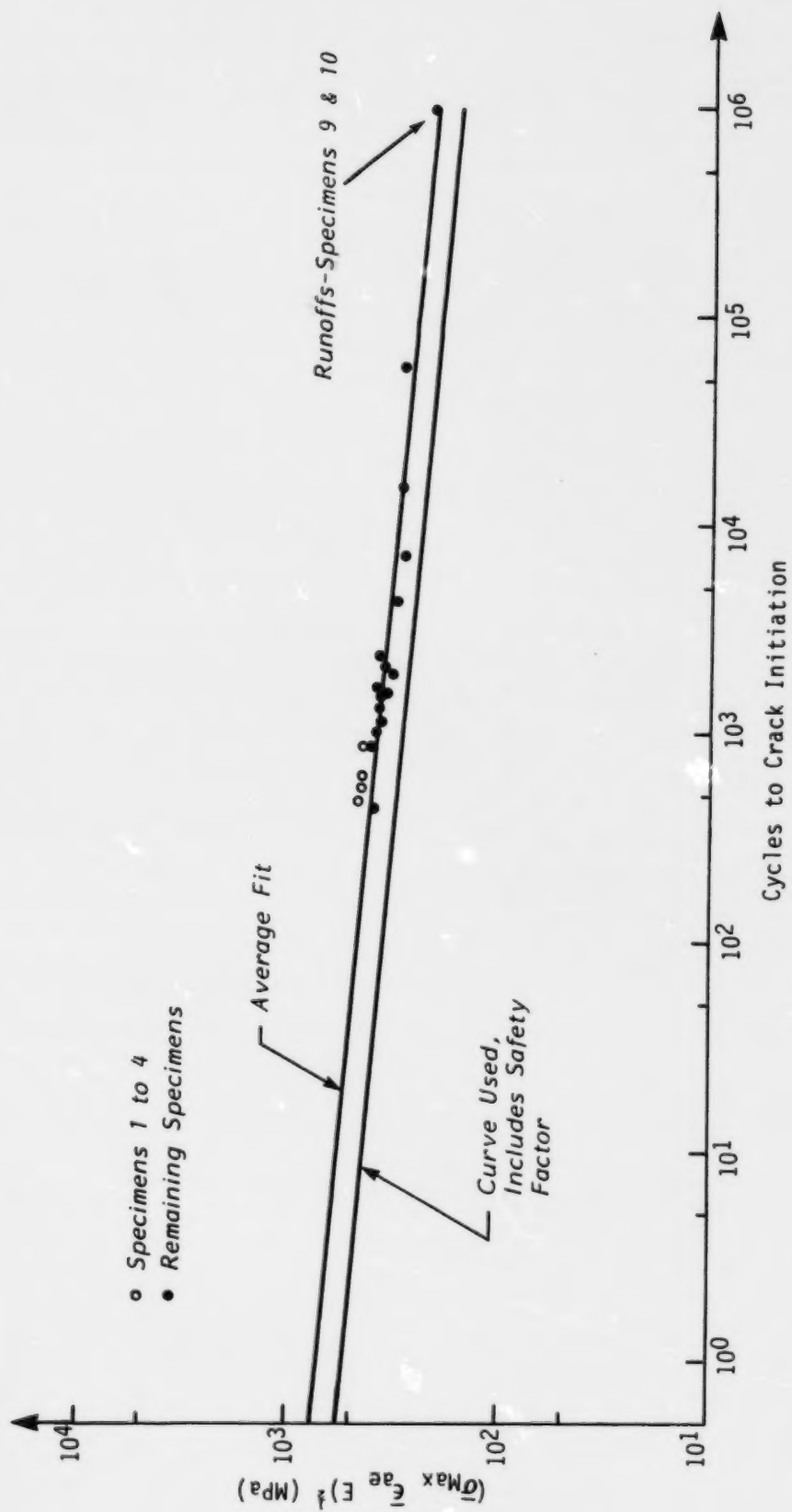


FIGURE 3-3
ELASTIC STRESS/STRAIN FATIGUE FUNCTION
DATA FROM EXPERIMENT

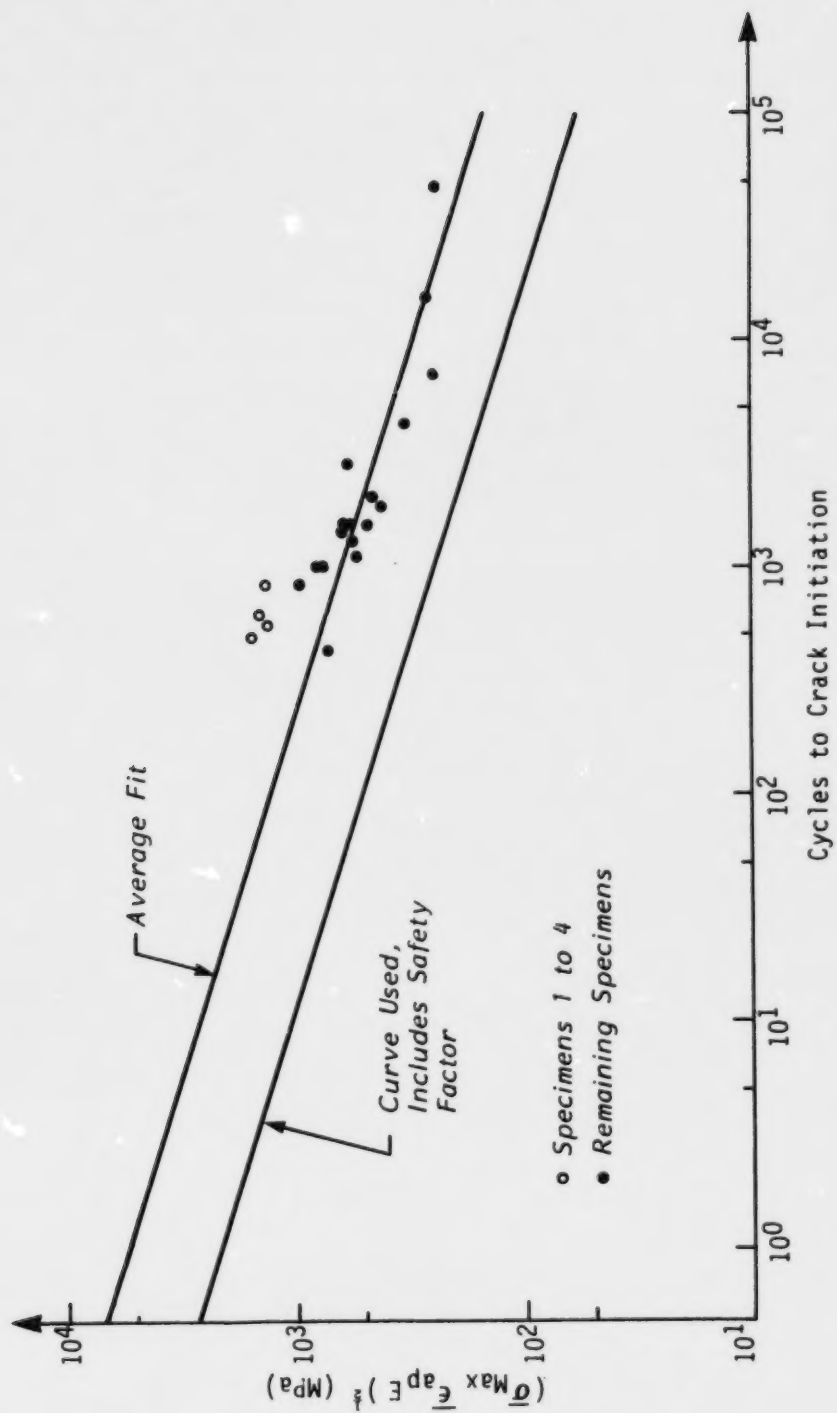


FIGURE 3-4
PLASTIC STRESS/STRAIN FATIGUE FUNCTION
DATA FROM EXPERIMENT

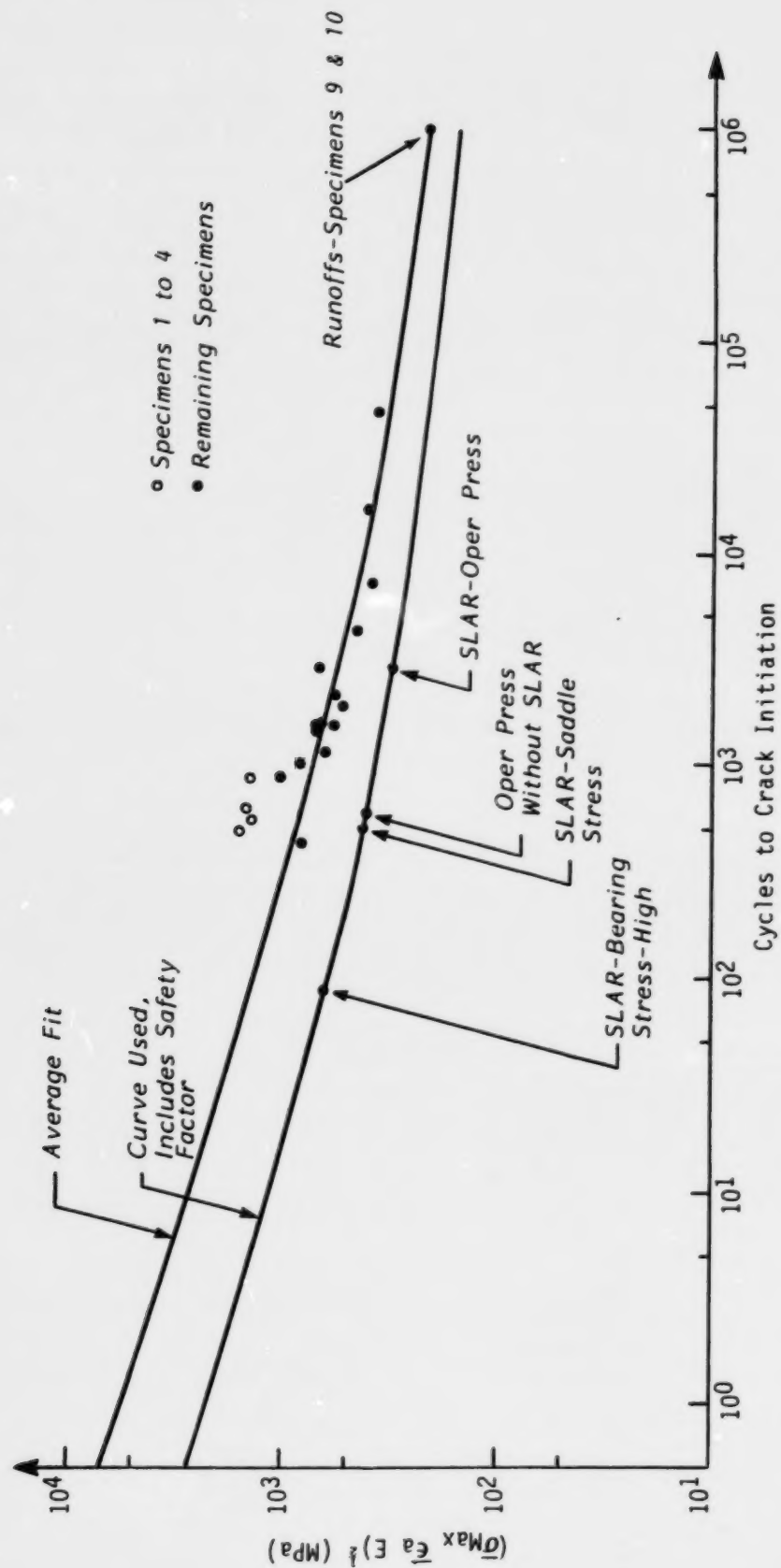


FIGURE 3-5
TOTAL STRESS/STRAIN FATIGUE DATA FROM EXPERIMENT, AND
POINTS USED IN CALCULATING ALLOWABLE FLAW ROOT RADII

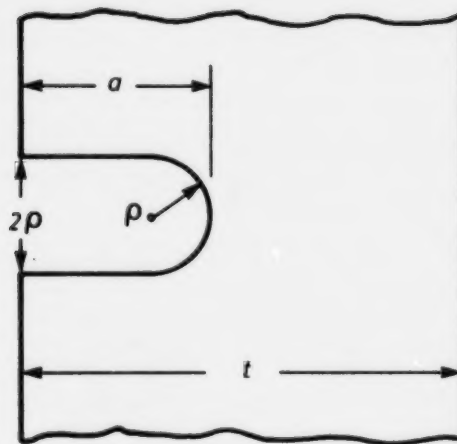


FIGURE 4-1
GEOMETRY OF U-NOTCH

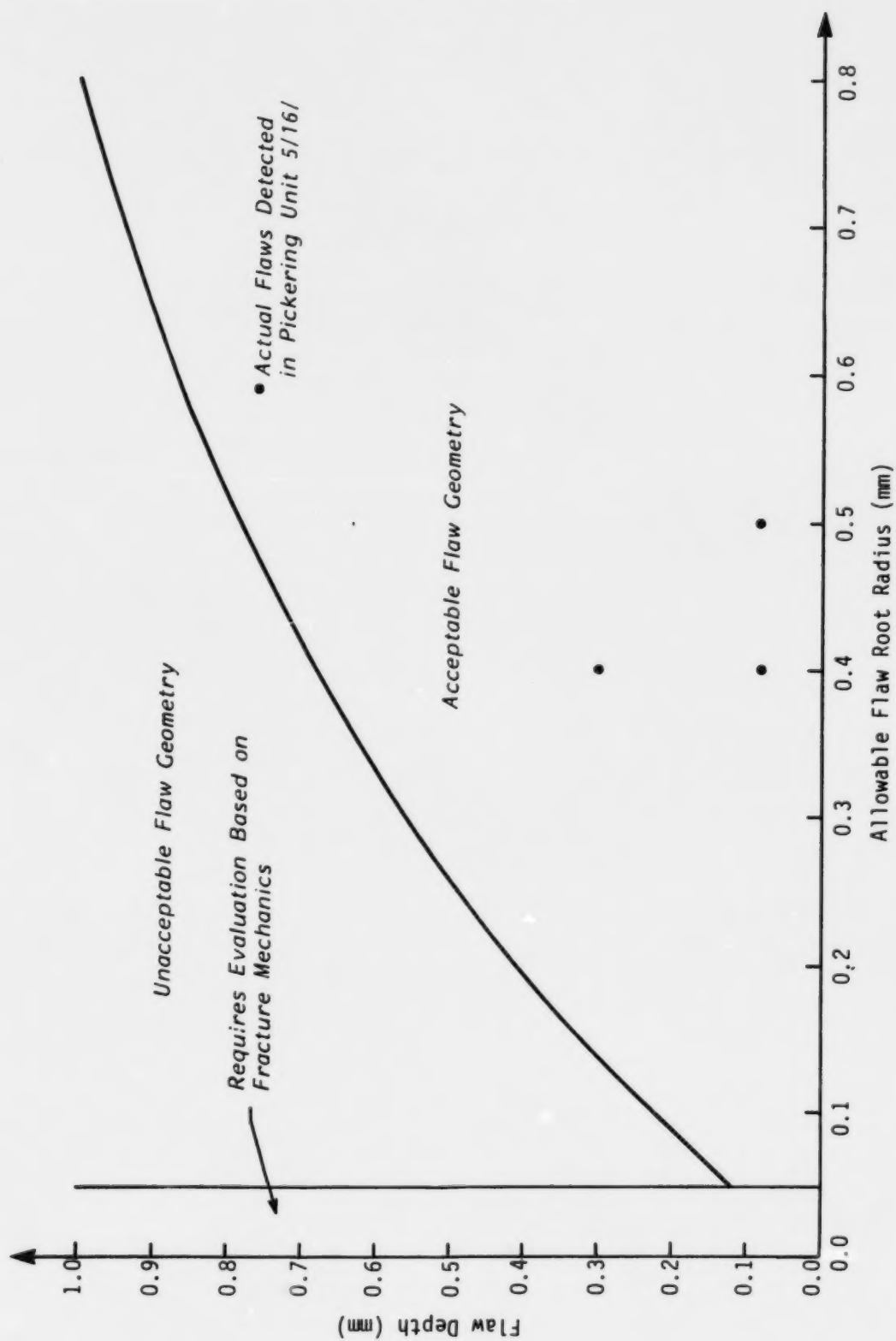


FIGURE 4-2
ALLOWABLE FLAW ROOT RADII FOR SLAR PLUS
NORMAL OPERATING STRESSES

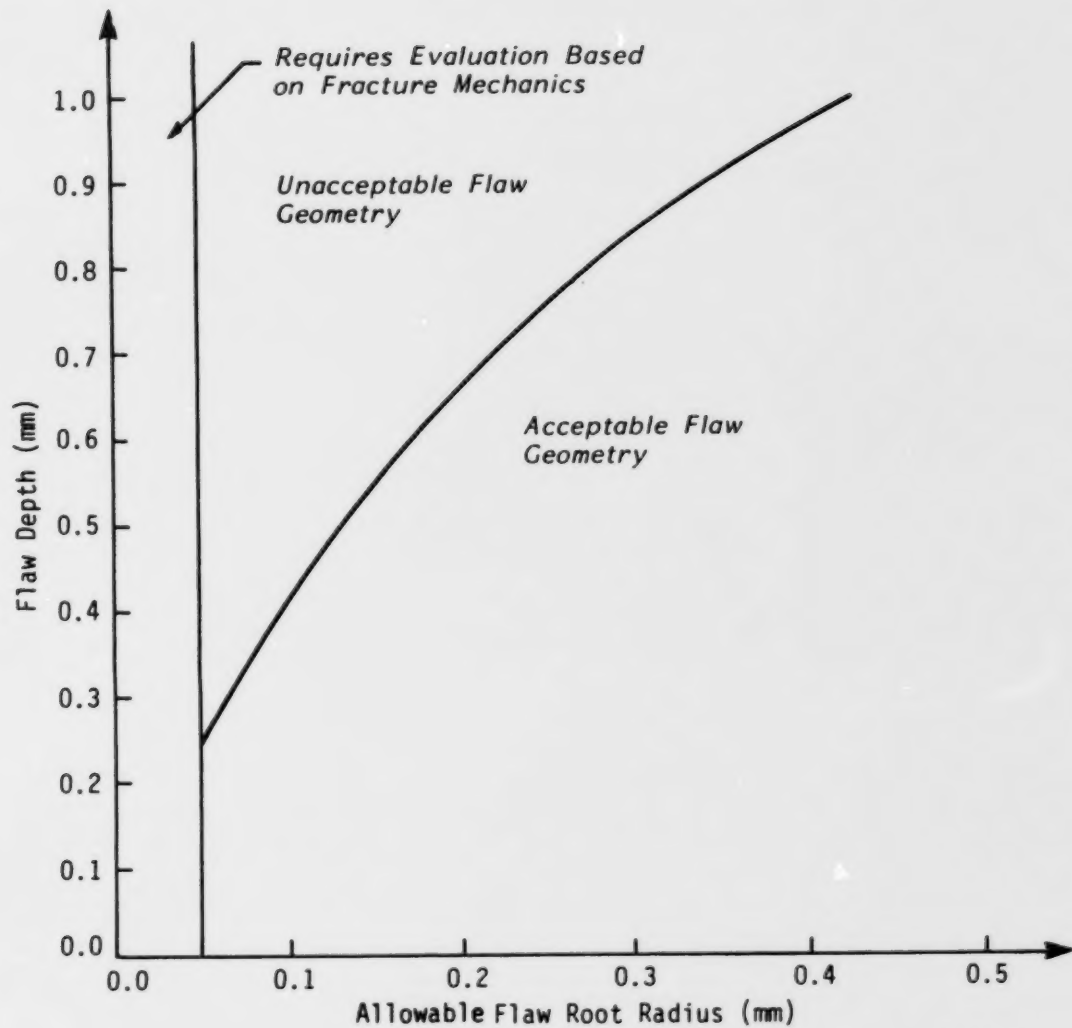


FIGURE 4-3
ALLOWABLE FLAW ROOT RADII FOR
NORMAL OPERATING STRESSES ONLY

APPENDIX A

Details of Cyclic Stress/Strain Calculations and Fatigue Usage Factors

A-1 Constitutive Model

The material constitutive relations are written in terms of cyclic changes in stress and strain. Deformation theory plasticity/18/ is employed in modelling the cyclic histories of the stress and strain components. Implicit is the assumption that a material point is subjected to proportional stressing. If expressed in terms of total stress and strain, deformation theory plasticity is only valid up to the point at which unloading occurs. However, if the deformation plasticity model is written in terms of cyclic ranges in stress and strain, it can be applied to model cyclic loading. Any unloading on the stress/strain plot is associated with positive values of cyclic changes in stress and strain. A comparison given in reference /19/ showed that for the case of a notched bar under uniaxial cyclic loading the deformation plasticity model, implemented into a finite element computer code, predicted cyclic strains at the notch tip which were comparable to results obtained by using a more detailed constitutive model for cyclic plasticity.

The material's stress/strain curve is idealized by the Ramberg-Osgood equation:

$$\Delta_T \bar{\epsilon} = \frac{\Delta_T \bar{\sigma}}{E} + \lambda \alpha \epsilon_0 \cdot \left(\frac{\Delta_T \bar{\sigma}}{\lambda \sigma_0} \right)^n \quad (A-1)$$

where $\Delta_T \bar{\epsilon}$ = effective cyclic total strain range,
 $\Delta_T \bar{\sigma}$ = effective cyclic total stress range,
 E = Young's modulus,
 σ_0 = yield stress,
 ϵ_0 = yield strain, = σ_0/E ,
 α = Ramberg-Osgood material constant,
 n = Ramberg-Osgood material strain hardening exponent,
 λ = multiplier to model the full stress/strain hysteresis loop.
 λ = 1.0 for first 1/2 cycle,
 λ = 2.0 for all subsequent reversals.

It is assumed in this analysis that the shear stresses and strains can be neglected. If no shear strains are present the cyclic range in effective strain is given by:

$$\Delta_T \bar{\epsilon} = \frac{\sqrt{2}}{3} [(\Delta_T \epsilon_{xx} - \Delta_T \epsilon_{yy})^2 + (\Delta_T \epsilon_{xx} - \Delta_T \epsilon_{zz})^2 + (\Delta_T \epsilon_{yy} - \Delta_T \epsilon_{zz})^2]^{1/2} \quad (A-2)$$

where ΔT^{ϵ}_{xx} - x-component of the cyclic total range in normal strain,
 ΔT^{ϵ}_{yy} - y-component of the cyclic total range in normal strain,
 ΔT^{ϵ}_{zz} - z-component of the cyclic total range in normal strain.

The notch geometry and local coordinate system is given in Figure A-1. This definition of effective strain was used since in a simple tension test with perfectly plastic behaviour (Poisson's ratio = 0.50) equation (A-2) reduces to $\Delta T^{\epsilon} = \Delta T^{\epsilon}_{xx}$.

If no shear stresses are present the effective cyclic total range in stress is given by:

$$\Delta T^{\sigma} = \frac{1}{\sqrt{2}} [(\Delta T^{\sigma}_{xx} - \Delta T^{\sigma}_{yy})^2 + (\Delta T^{\sigma}_{xx} - \Delta T^{\sigma}_{zz})^2 + (\Delta T^{\sigma}_{yy} - \Delta T^{\sigma}_{zz})^2]^{1/2} \quad (A-3)$$

where ΔT^{σ}_{xx} - x-component of the cyclic total range in normal stress, etc.

By using deformation theory plasticity^{18/} the relation between cyclic ranges in stress and plastic strain components is:

$$\Delta T^{\epsilon}_{ijp} = \frac{3}{2} \frac{\Delta T^{\epsilon}_p}{\Delta T^{\sigma}} \Delta T^D_{ij} \quad (A-4)$$

where $\Delta T^{\epsilon}_{ijp}$ - cyclic total range in plastic strain component ij,
 ΔT^{ϵ}_p - effective cyclic total range in plastic strain,
 ΔT^D_{ij} - cyclic total range in deviatoric stress.

By substituting the second term of equation (A-1) into (A-4), and adding the elastic strains in terms of the generalized Hooke's law, the constitutive relations between cyclic ranges in stress and strain components are:

$$\Delta T^{\epsilon}_{xx} = - \left(\frac{\nu}{E} + C_1 \Delta T^{\sigma n-1} \right) (\Delta T^{\sigma}_{yy} + \Delta T^{\sigma}_{zz}) \quad (A-5)$$

$$\Delta T^{\epsilon}_{yy} = \frac{1}{E} (\Delta T^{\sigma}_{yy} - \nu \Delta T^{\sigma}_{zz}) + C_1 \Delta T^{\sigma n-1} (2\Delta T^{\sigma}_{yy} - \Delta T^{\sigma}_{zz}) \quad (A-6)$$

$$\Delta T^{\epsilon}_{zz} = \frac{1}{E} (\Delta T^{\sigma}_{zz} - \nu \Delta T^{\sigma}_{yy}) + C_1 \Delta T^{\sigma n-1} (2\Delta T^{\sigma}_{zz} - \Delta T^{\sigma}_{yy}) \quad (A-7)$$

$$C_1 = \frac{\alpha}{2E (\lambda \sigma_0)^{n-1}} \quad (A-8)$$

where $\sigma_{xx} = 0$ at the notch surface,
 ν - Poisson's ratio.

The use of the Ramberg-Osgood equation (A-1) is illustrated in Figure A-2. In this figure the idealized effective stress/effective strain curves are plotted. The curve labeled $\lambda = 1$ corresponds to the initial loading of the material, while the curves labeled $\lambda = 2$ are amplified by a factor of 2 and model the hysteresis loop associated with the stress reversals. The origins of the $\lambda = 2$ curves are assumed to lie at $\bar{\sigma} = +\sigma_u$ and $\bar{\sigma} = -\sigma_u$ respectively, where σ_u is the material's ultimate tensile stress. Referring to Figure A-2, during a one-half cycle the effective stress would change from $+\bar{\sigma}_1$ to $-\bar{\sigma}_2$ with an actual amplitude of $\Delta\bar{\sigma}$. However, the nonlinear Ramberg-Osgood equation must be expressed in terms of the total stress amplitude $\Delta_T\bar{\sigma}$ which originates from the shifted origin at $\bar{\sigma} = +\sigma_u$. The $\Delta_T\bar{\sigma}$ contains a fictitious quantity $\bar{\sigma}_d$ which is the difference between the total $\Delta_T\bar{\sigma}$ and the actual $\Delta\bar{\sigma}$. Once the total range in stress components $\Delta_T\sigma_{ij}$ are calculated, the actual range in stress components is calculated from:

$$\Delta\sigma_{ij} = \Delta_T\sigma_{ij} - \sigma_{ij_d} \quad (A-9)$$

The components of σ_{ij_d} are determined by using the assumption of proportional stressing. Under this assumption the "shape" of the tensor σ_{ij_d} will be the same as the previous $\Delta_T\sigma_{ij}$. Therefore, let:

$$\sigma_{ij_d} = \phi \Delta_T\sigma_{ij} \quad (A-10)$$

where:

$$\phi = \frac{(\sigma_u - \bar{\sigma}_1)}{\Delta_T\bar{\sigma}} \quad (A-11)$$

The total range in strain components are calculated by using equations (A-5) to (A-7). The total and actual range in strain components are related by:

$$\Delta\epsilon_{ij} = \Delta_T\epsilon_{ij} - \epsilon_{ij_d} \quad (A-12)$$

where ϵ_{ij_d} are calculated by substituting σ_{ij_d} into equations (A-5) to (A-7).

A-2 Notch-Tip Stress/Strain Concentration Factors

A-2.1 Elastic Stress Concentration Factors

The nominal stress distribution through the wall thickness of the pressure tube is idealized as having a membrane component, σ_m , and a bending component, σ_b , as illustrated in Figure A-3. These stresses are normalized by dividing by the inside wall stress, σ_{in} .

$$\bar{\sigma}_m = \frac{\sigma_m}{\sigma_{in}} \quad (A-13)$$

$$\bar{\sigma}_b = \frac{\sigma_b}{\sigma_{in}} \quad (A-14)$$

The centre of application of the remote membrane stress is offset from the centroid of the net section ligament at the notch. A bending moment is therefore induced across the net ligament by the remote membrane stress.

The elastic notch-tip stress concentration factors based on pure membrane and pure bending stresses across the net ligament are denoted as k_m and k_b , respectively. These will be defined below for each notch type. The elastic notch-tip stress concentration factor due to the combined remote membrane and bending stresses is given by:

$$k_t = \frac{1}{(1-a/t)} [k_m \bar{\sigma}_m + k_b (\frac{3\bar{\sigma}_m}{(t/a-1)} + \frac{\bar{\sigma}_b}{(1-a/t)})] \quad (A-15)$$

where "a" is the notch depth and t is the wall thickness. The stress concentration factor k_t is defined as the normal stress σ_{yy} at the notch root divided by the inside wall stress, σ_{in} .

A-2.1.1 Elastic Notch-Tip Stress Concentration Factors for the V-Notch

V-notches with rounded roots were used in the fatigue experiments/6/.

Due to Membrane Stress:

The solution for a notch in a plate of finite thickness under a pure membrane stress was not known to the author. The solution for a notch in a plate with semi-infinite thickness was judged to be adequate and is given in Figure A-4, which was taken from reference /21/.

Due to Bending Stress:

The following equation is for a finite thickness plate under pure bending and was taken from reference /20/.

k_b is the lesser of k_{bU} or

$$1.11 k_{bU} - 0.0304 k_{bU}^2 \quad (A-16)$$

where k_{bU} is the elastic notch-tip stress concentration factor for a U-notch in pure bending, given in Section A-2.1.2.

A-2.1.2 Elastic Notch-Tip Stress Concentration Factors for the U-Notch

Solutions were taken from reference /20/ for a notch in a finite-thickness plate. The postulated flaws in the pressure tube are U-notches.

Due to Membrane Stress:

$$k_m = A_1 + A_2 a/t + A_3(a/t)^2 + A_4(a/t)^3$$

$$\text{where } A_1 = 0.721 + 2.394 (a/\rho)^{1/2} - 0.127 a/\rho$$

$$A_2 = 1.978 - 11.489 (a/\rho)^{1/2} + 2.211 a/\rho$$

$$A_3 = -4.413 + 18.751 (a/\rho)^{1/2} - 4.596 a/\rho$$

$$A_4 = 2.714 - 9.655 (a/\rho)^{1/2} + 2.512 a/\rho \quad (A-17)$$

$$\text{and } 0.5 \leq a/\rho \leq 4.0,$$

where ρ is the root radius.

Due to Bending Stress:

$$k_b = B_1 + B_2 a/t + B_3(a/t)^2 + B_4(a/t)^3$$

$$\text{where } B_1 = 0.721 + 2.394 (a/\rho)^{1/2} - 0.127 a/\rho$$

$$B_2 = -0.426 - 8.827 (a/\rho)^{1/2} + 1.518 a/\rho$$

$$B_3 = 2.161 + 10.968 (a/\rho)^{1/2} - 2.455 a/\rho$$

$$B_4 = -1.456 - 4.535 (a/\rho)^{1/2} + 1.064 a/\rho \quad (A-18)$$

A-2.2 Elastic-Plastic Stress/Strain Concentration Factors

The notch-tip stress concentration factor, k_σ , and the notch-tip strain concentration factor, k_ϵ , for a notch in a pure power-law hardening material under uniaxial shear are related by Neuber's equation /22/:

$$k_t = (k_\sigma k_\epsilon)^{1/2} \quad (A-19)$$

where k_t is the elastic notch-tip stress concentration factor. Equation (A-19) is applied to relate the vertical components (y-components in Figure A-1) of cyclic ranges in stress and strain/9/. By denoting the nominal inside wall stress range as ΔS_{yy} , equation (A-19) can be rewritten as:

$$\Delta \epsilon_{yy} = \frac{(k_t \Delta S_{yy})^2}{E \Delta \sigma_{yy}} \quad (A-20)$$

where the elastic notch-tip stress concentration factor k_t is based on the nominal inside wall stress, σ_{in} .

A-3 Cyclic Stress/Strain Tracing

Since deformation theory plasticity is used, the strains are uniquely related to the stresses through the constitutive relations given in Section A-1. Therefore, the cyclic range in stress components only will be determined first. The normal stress σ_{xx} (see Figure A-1) is zero at the notch root. It is assumed that the shear stresses are not significant. The first of the two equations required to solve for the total range in the stress components $\Delta_T \sigma_{yy}$ and $\Delta_T \sigma_{zz}$ is obtained by combining Neuber's relation (Equation A-19), the constitutive relation for the total strain range $\Delta_T \epsilon_{yy}$ at the notch root (Equation A-6), and Equation (A-12),

$$\begin{aligned} & \frac{(k_t \Delta S_{yy})^2}{(\Delta_T \sigma_{yy} - \sigma_{yyd})} + E \epsilon_{yyd} = \Delta_T \sigma_{yy} - \nu \Delta_T \sigma_{zz} \\ & + \frac{\alpha}{2 (\lambda \sigma_0)^{n-1}} \Delta_T \bar{\sigma}(\Delta_T \sigma_{yy}, \Delta_T \sigma_{zz})^{n-1} (2 \Delta_T \sigma_{yy} - \Delta_T \sigma_{zz}) \end{aligned} \quad (A-21)$$

where $\Delta_T \bar{\sigma}(\)$ denotes a functional dependence.

The second equation is based on the assumption that, in the absence of any applied longitudinal stress $\Delta \sigma_{zz}$, plane strain conditions exist at the notch root and the cyclic strain component $\Delta \epsilon_{zz}$ is zero. This assumption is used to define a portion of the z-component of stress range, denoted as $\Delta \sigma_{zz-p}$, which is the reaction to the vertical stress $\Delta \sigma_{yy}$ due to the Poisson effect. Any remote nominal longitudinal stress range, denoted as $\Delta \sigma_{zz-r}$, is added on to $\Delta \sigma_{zz-p}$ to define the total stress range $\Delta \sigma_{zz}$. Under these assumptions equation (A-7) becomes:

$$\begin{aligned} & E \epsilon_{zzd} = \Delta_T \sigma_{zz} - \Delta \sigma_{zz-r} - \nu \Delta_T \sigma_{yy} \\ & + \frac{\alpha}{2 (\lambda \sigma_0)^{n-1}} \Delta_T \bar{\sigma}(\Delta_T \sigma_{yy}, \Delta_T \sigma_{zz} - \Delta \sigma_{zz-r})^{n-1} \\ & \times (2 (\Delta_T \sigma_{zz} - \Delta \sigma_{zz-r}) - \Delta_T \sigma_{yy}) \end{aligned} \quad (A-22)$$

Equations (A-21) and (A-22) are solved simultaneously for $\Delta_T \sigma_{yy}$ and $\Delta_T \sigma_{zz}$ by using Newton-Raphson iteration.

Once the cyclic total stress ranges $\Delta_T \sigma_{yy}$ and $\Delta_T \sigma_{zz}$ have been calculated, the actual ranges $\Delta \sigma_{yy}$ and $\Delta \sigma_{zz}$ are calculated by using equation (A-9). The actual cyclic range in strain components are then calculated by substituting $\Delta_T \sigma_{yy}$ and $\Delta_T \sigma_{zz}$ into equations (A-5) to (A-7), and by using equation (A-12).

The calculations are performed on a reversal-by-reversal basis. Referring to Figure A-5, the first reversal is 0-1, the second is 1-2, the third is 2-1, etc. The cyclic stress and strain components are alternately added and then subtracted with each reversal to the running totals of stress and strain.

A-4 Use of Effective Strain as a Fatigue Criterion

In a multi-axial stress state yielding is characterized by the value of the effective stress, $\bar{\sigma}$. The effective stress is written in terms of the deviatoric stresses, which are generally accepted as causing plastic deformation. An effective strain, given by equation (A-2), will be used to characterize the tri-axial strain state at the notch tip for fatigue crack initiation. This effective strain is written in terms of the deviatoric strains, which have also been used to characterize the strain state for fatigue analysis/9/.

A-5 Treatment of Load Combinations

The damage due to various levels of alternating strains applied for various numbers of cycles is treated by using the Miner cumulative damage rule /8,32/ applied to crack initiation. The damage d_j due to one value of the function $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)_j^{1/2}$ for N_j number of cycles is:

$$d_j = \frac{\text{cycles at } (\bar{\sigma}_{\max} \bar{\epsilon}_a E)_j^{1/2}}{\text{cycles to crack initiation at } (\bar{\sigma}_{\max} \bar{\epsilon}_a E)_j^{1/2}} \quad (\text{A-23})$$

$$d_j = \frac{N_j}{N_{fj}} \quad (\text{A-24})$$

where N_{fj} is the number of cycles to crack initiation at the value of $(\bar{\sigma}_{\max} \bar{\epsilon}_a E)_j^{1/2}$.

The values of d_j for various load cases sum up to a cumulative usage factor. In design the value of the usage factor cannot exceed 1.0. The limiting cumulative damage equation is:

$$\sum \frac{SF \cdot N_j}{N_{fj}} = 1.0 \quad (\text{A-25})$$

where SF is the safety factor on the number of cycles. A safety factor of 20 on number of cycles from the "best fit" experimental curve, used in the ASME Section III fatigue design curves/8/, has been used.

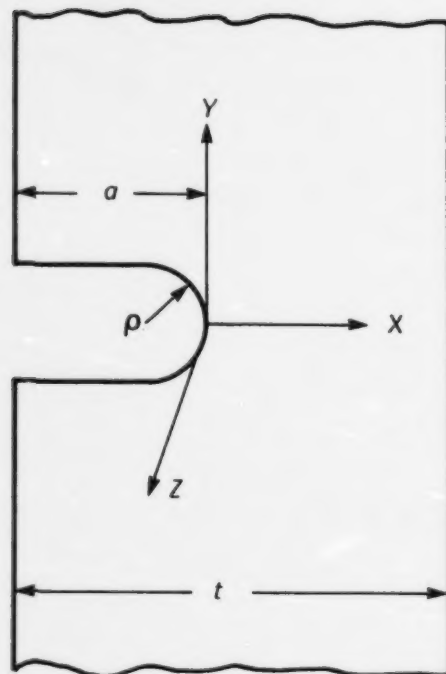


FIGURE A-1
NOTCH GEOMETRY AND LOCAL COORDINATE SYSTEM

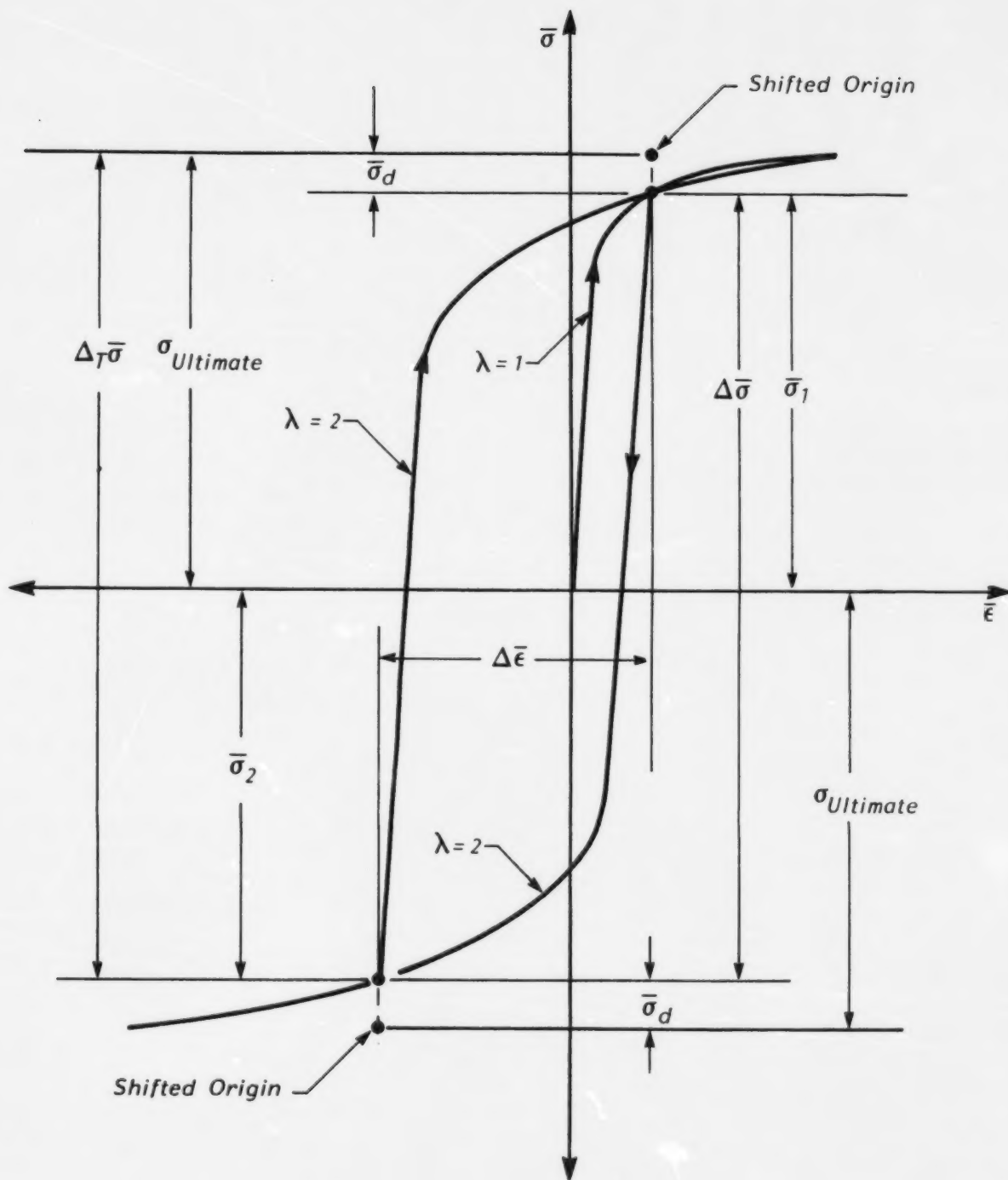


FIGURE A-2
ILLUSTRATION OF CYCLIC STRESS/STRAIN
TRACING FROM SHIFTED ORIGINS

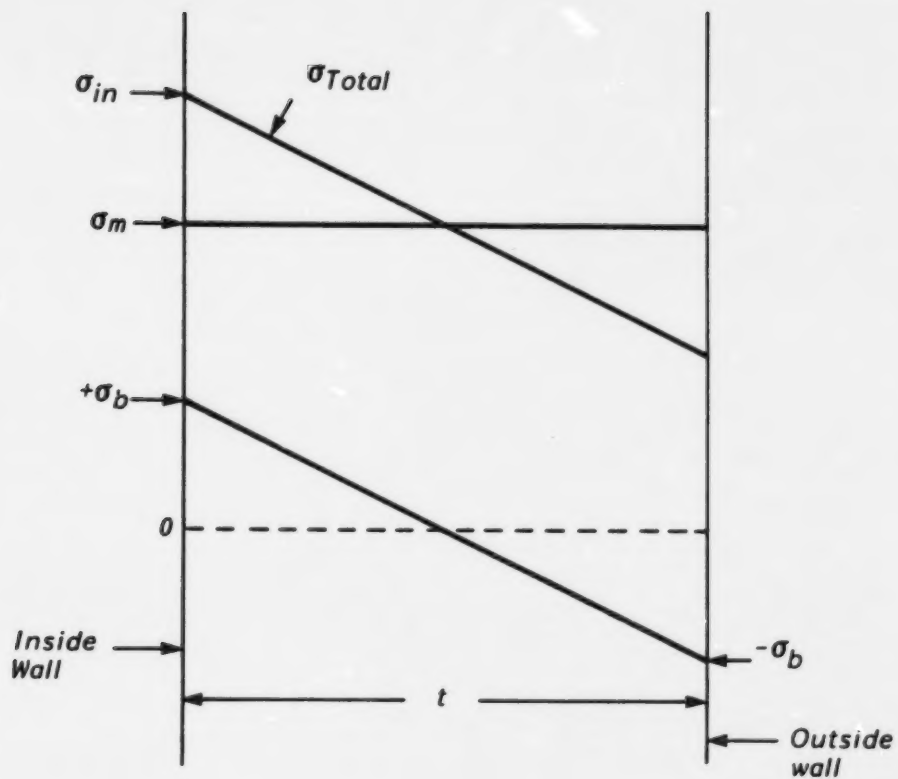


FIGURE A-3
ILLUSTRATION OF MEMBRANE AND
THROUGH-WALL BENDING COMPONENTS OF STRESS

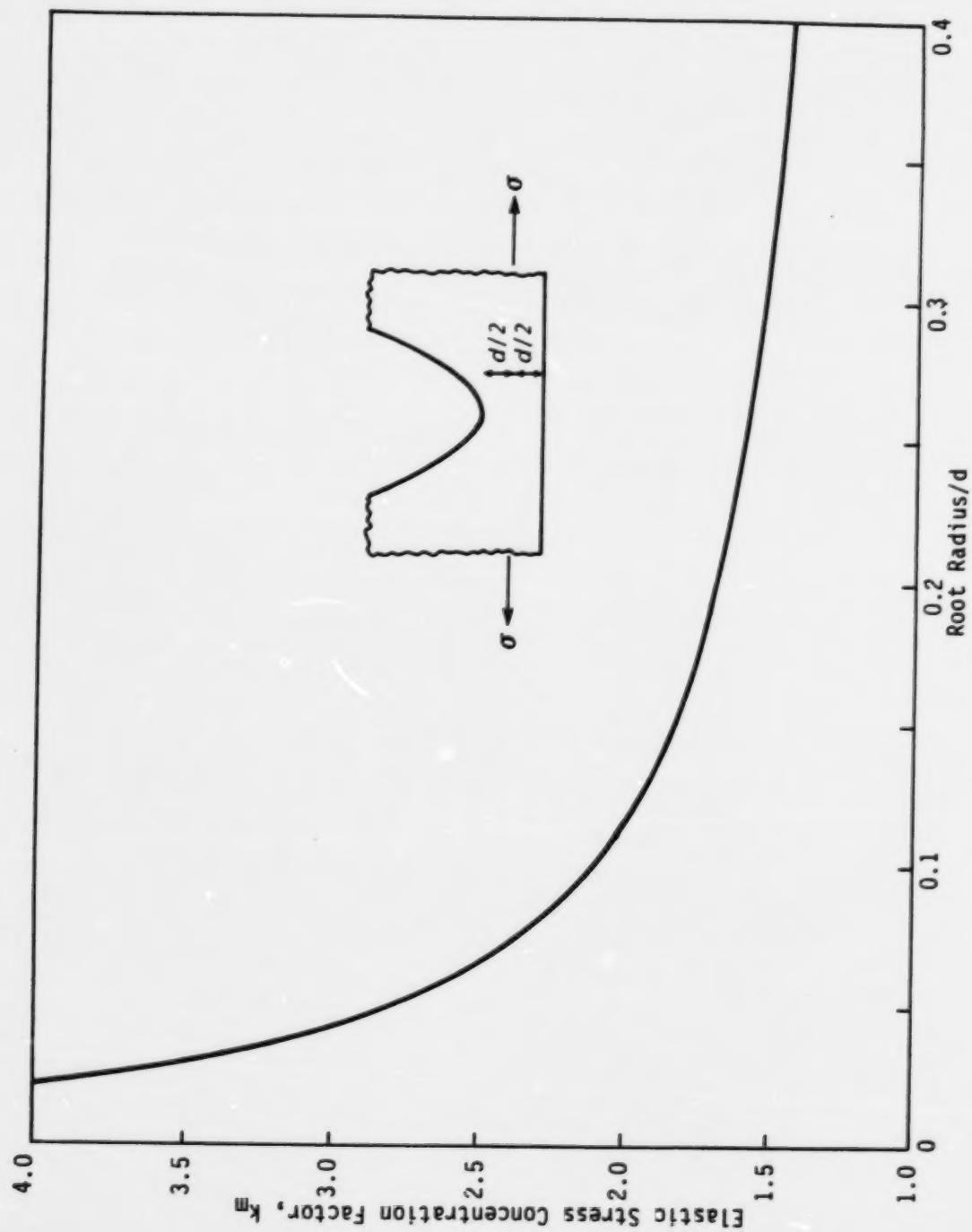


FIGURE A-4

ELASTIC STRESS CONCENTRATION FACTOR FOR A V-NOTCH UNDER A MEMBRANE STRESS CENTRED ON THE NET LIGAMENT (FIGURE 22 ON PAGE 40 OF REFERENCE/21/)

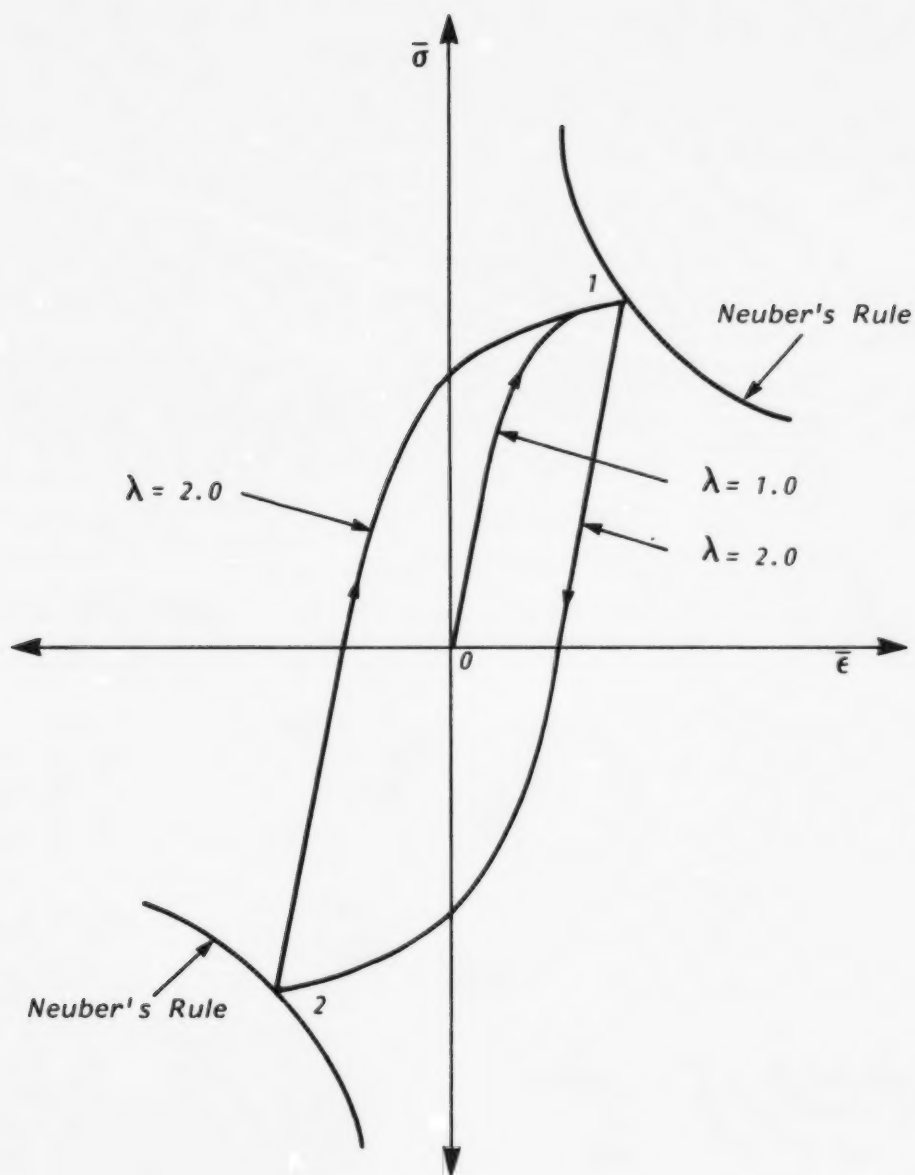


FIGURE A-5
ILLUSTRATION OF CYCLIC STRESS/STRAIN TRACING
USING NEUBER'S RULE

APPENDIX B

Calculation of Nominal Stresses

B-1 Nominal Stresses in Experimental Test Specimens

The nominal stress at the inside wall of the ring specimen is given by /13/:

$$\sigma_{in} = 2.4 \sigma_m + 28.3 \text{ MPa} \quad (\text{B-1})$$

where σ_m is the membrane hoop stress. The normalized membrane and bending hoop stresses (see Figure A-3) in the experiment are:

$$\bar{\sigma}_m = \frac{\sigma_m}{\sigma_{in}} \quad (\text{B-2})$$

$$\bar{\sigma}_b = 1.0 - \bar{\sigma}_m \quad (\text{B-3})$$

B-2 Nominal Stresses in the Pressure Tube

B-2.1 Nominal Stresses Due to the SLAR Operation

The stresses and number of cycles are given in Table 4-1. The stresses used were at the circumferential location of the maximum inside wall hoop stress, at the 3 o'clock position. As recommended in reference /2/, the stresses calculated in the finite element analysis were increased by 30% to account for discrepancies between the finite element calculations and results obtained from strain-gauged experiments performed with the SLAR tool in a pressure tube.

To calculate the membrane and through-wall bending hoop stresses, the hoop stress at the inside and outside wall surfaces must be used. However, only the axial profile of the inside wall stress is currently available at the 3 o'clock position. A conservative estimate will be made of the membrane and through-wall bending components of hoop stress. The axial profile of inside wall hoop stress will be used to determine the number of SLAR stress cycles.

B-2.1.1 Estimates of Number of SLAR Load Cycles

The number of cycles, N_i , which a given axial stress profile of half-length L_i is predicted to act on the pressure tube is given by:

$$N_i = \frac{2 \times (\text{no. of garter springs}) \times (\text{no. of reaction points}) \times L_i}{\text{movement of garter spring during one cycle}} \quad (\text{B-4})$$

It is assumed that the number of garter springs which must be moved is 4. The movement of the garter spring during one SLAR load cycle is taken to be 13 mm (0.5")/2/. The number of reaction points for the saddles is 1, since the stresses due to the three saddles were combined by linear superposition/2/. The number of reaction points for the bearings is 2. The number of cycles can then be written as:

for the saddles,

$$N_1 = 0.630 L_1 \quad (B-5)$$

and for the bearings,

$$N_1 = 1.260 L_1 \quad (B-6)$$

where L_1 is in mm.

B-2.1.2 Nominal Hoop Stress Due to the Saddles

The axial profile of hoop stress at the inside wall was taken from /3/ and is reproduced in Figure B-1. This represents the superposition of stresses due to all three saddles. The stress envelope used in determining the allowable root radii is also shown, with a stress level of 211 MPa. The length of the stress envelope was taken to be one-half of the maximum length over which significant stresses would exist due to the array of the 3 saddles. This half-length is 356 mm. From equation (B-5), the number of stress cycles was estimated at 224.

At the axial location of contact of the saddle or bearing at the 3 o'clock position the hoop stress is tension on the inside wall and compression on the outside wall. Thus the through-wall bending component of hoop stress is greater than the membrane component at that location. The following distribution will be assumed for the membrane and through-wall bending components of hoop stress at all axial locations:

$$\bar{\sigma}_m = 0.50 \quad (B-7)$$

$$\bar{\sigma}_b = 0.50 \quad (B-8)$$

B-2.1.3 Nominal Longitudinal Stress Due to the Saddles

The longitudinal stress at the axial location of contact of a saddle at the 3 o'clock position varies between +90 MPa at the inside surface and -90 MPa at the outside surface. The axial variation of longitudinal stress at the outside wall is not available at the 3 o'clock position. Test calculations showed that a positive longitudinal stress results in allowable root radii which are slightly larger than that using a zero or negative longitudinal stress. Therefore a longitudinal stress of +90 MPa was assumed to act in a pure membrane mode for 224 cycles.

B-2.1.4 Nominal Hoop Stress Due to the Bearings

The axial profile of hoop stress at the 3 o'clock position at the inside wall was taken from reference /4/ and is reproduced in Figure B-2. This stress profile was calculated by finite element analysis for a bearing with a 30" crown radius. The bearing crown radius of the SLAR tool was later increased to 80" to reduce the peak hoop stress due to the bearing by about 20%/4/. The peak hoop stress occurred at the 6 o'clock position on the outside surface. It is not clear what reduction in hoop stress occurs on the inside surface at the 3 o'clock position. Therefore, the stress envelope shown in Figure B-2 was based on the results for the 30" crown radius bearing. The stress envelope is comprised of three sections with stress levels of 305, 152 and 90 MPa. The profile half-lengths of 25, 38 and 90 mm result in predicted numbers of cycles of 32, 48 and 112, respectively.

The assumed membrane and through-wall bending components of hoop stress are given by equations (B-7) and (B-8).

B-2.1.5 Nominal Longitudinal Stress Due to the Bearings

The longitudinal stress at the axial location of contact of a bearing at the 3 o'clock position varies between +90 MPa on the inside wall surface and -152 MPa on the outside wall surface. The axial variation of the longitudinal stress at the outside wall is not available at the 3 o'clock position. For reasons given in Section B-2.1.3, positive longitudinal stresses of magnitudes 90, 90 and 45 MPa were assumed to act in a pure membrane mode corresponding to the three stress envelope sections shown in Figure B-2. The number of stress cycles was assumed to be 32, 48 and 112, respectively.

B-2.2 Nominal Stresses Due to Operating Pressure

Stresses were calculated for an operating pressure of 10.0 MPa. A total of 550 startup/shutdown cycles was assumed.

B-2.2.1 Nominal Hoop Stress Due to Operating Pressure

The distribution of hoop stress through the wall of an internally pressurized cylinder is given in reference /23/ as:

$$\sigma_{\theta\theta}(r) = \frac{p R_i^2}{(R_o^2 - R_i^2)} \cdot \left[1 + \left(\frac{R_o}{r} \right)^2 \right] \quad (B-9)$$

where p is internal pressure, r the radius from the centre of the cylinder, R_i the radius at the inside wall and R_o the radius at the outside wall. The

normalized membrane and through-wall bending stress components are calculated by using:

$$\bar{\sigma}_m = \frac{1}{2} (1 + \sigma_{out}/\sigma_{in}) \quad (B-10)$$

$$\bar{\sigma}_b = \frac{1}{2} (1 - \sigma_{out}/\sigma_{in}) \quad (B-11)$$

where σ_{out} is the hoop stress at the outside wall and the normalization is by dividing by σ_{in} .

B-2.2.2 Nominal Longitudinal Stress Due to Operating Pressure

The longitudinal stress is a uniform membrane stress through the wall thickness and is given by:

$$\sigma_{zz-r} = \frac{p R_i}{2t} \quad (B-12)$$

where t is the wall thickness.

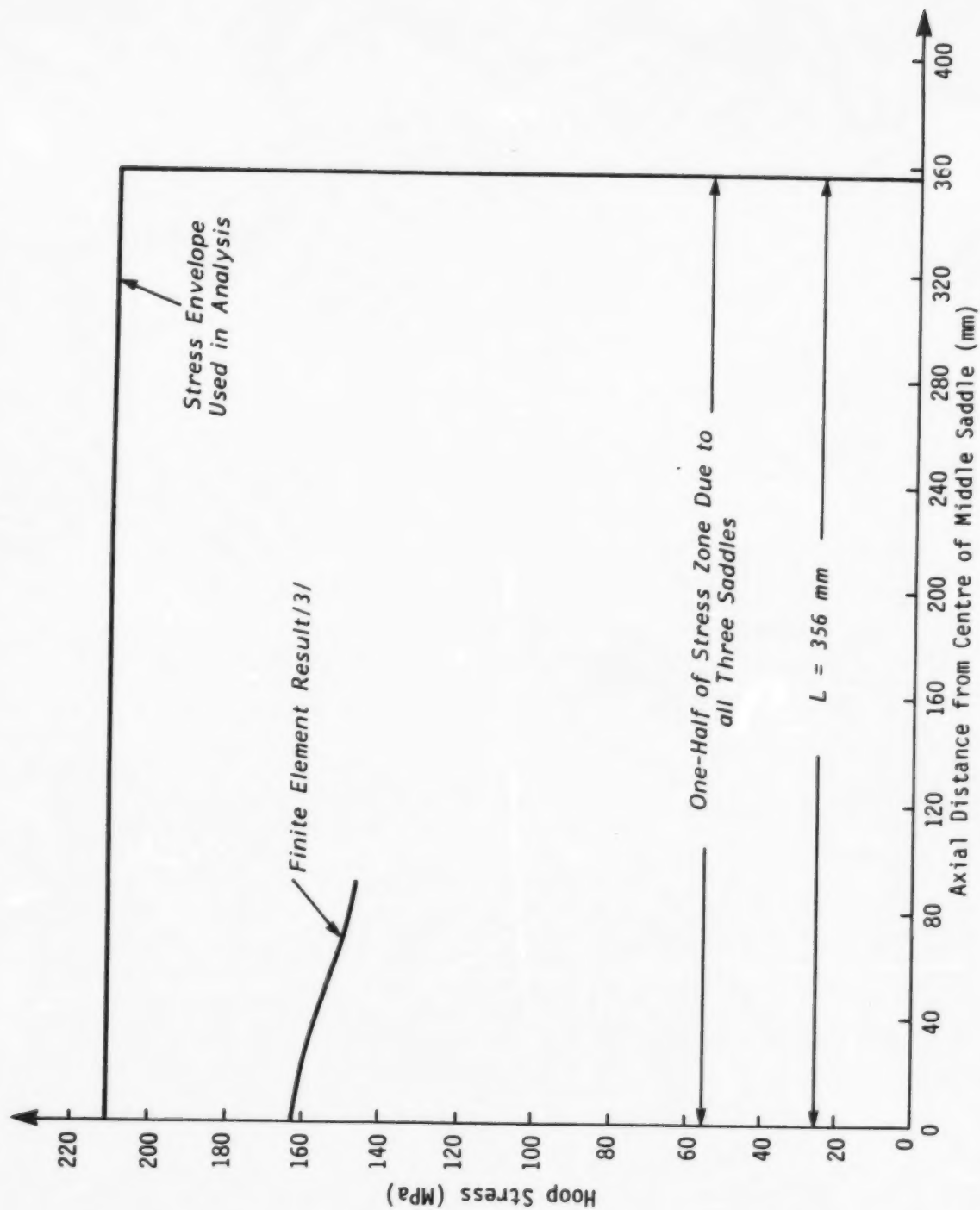


FIGURE B-1

AXIAL VARIATION OF THE HOOP STRESS ON THE
INSIDE WALL AT THE 3 O'CLOCK POSITION DUE TO THE SADDLES

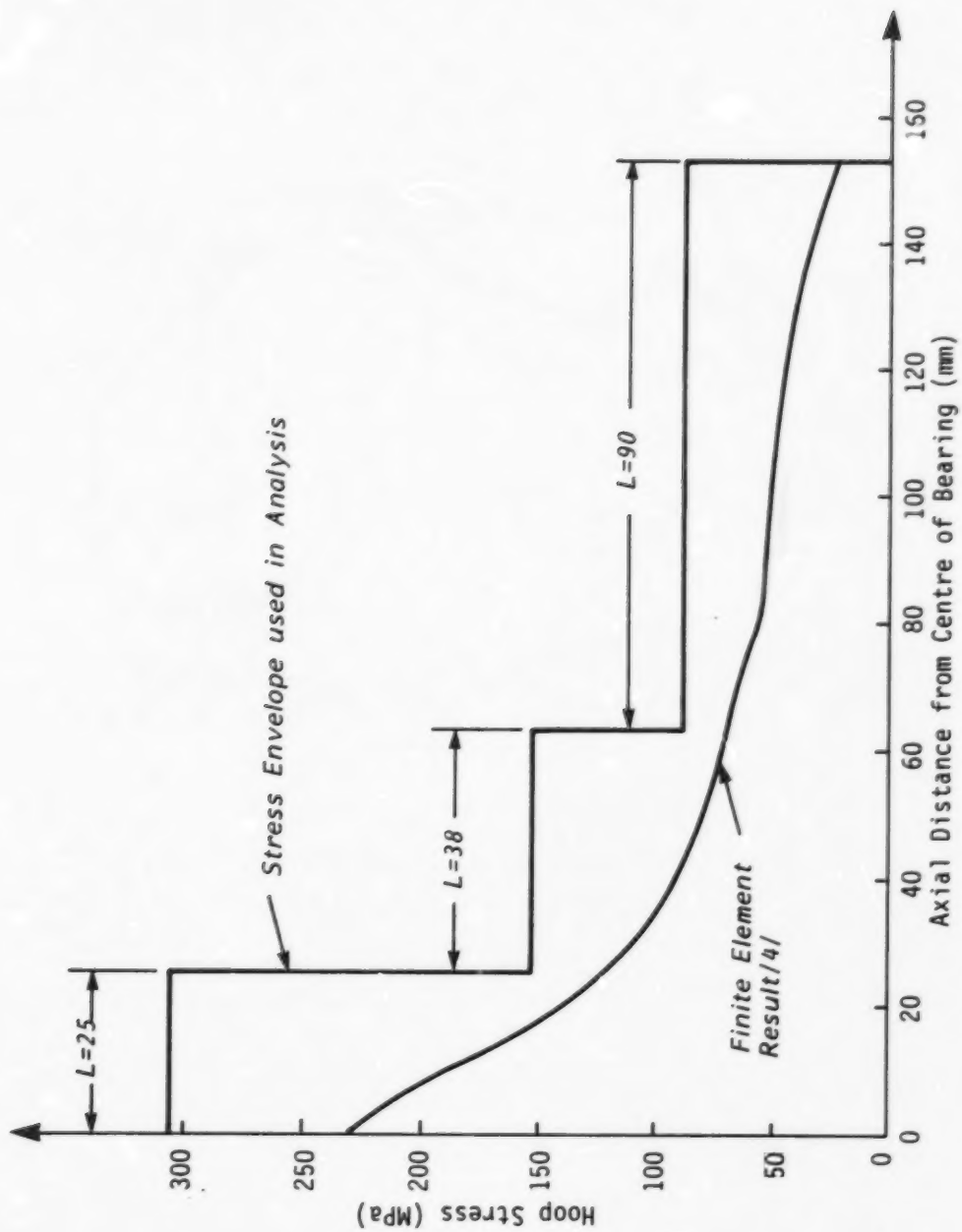


FIGURE B-2

AXIAL VARIATION OF THE HOOP STRESS ON THE INSIDE WALL
AT THE 3 O'CLOCK POSITION DUE TO THE BEARING